इंटरनेट मानक

Disclosure to Promote the Right To Information

Whereas the Parliament of India has set out to provide a practical regime of right to information for citizens to secure access to information under the control of public authorities, in order to promote transparency and accountability in the working of every public authority, and whereas the attached publication of the Bureau of Indian Standards is of particular interest to the public, particularly disadvantaged communities and those engaged in the pursuit of education and knowledge, the attached public safety standard is made available to promote the timely dissemination of this information in an accurate manner to the public.

“जानने का अधिकार, जीने का अधिकार”
Mazdoor Kisan Shakti Sangathan
“The Right to Information, The Right to Live”

“पुराने को छोड नये के तरफ”
Jawaharlal Nehru
“Step Out From the Old to the New”

IS 1548 (1981): Manual on Basic Principles of Lot Sampling
Itemized lot sampling [MSD 3: Statistical Methods for Quality and Reliability]

“ज्ञान से एक नये भारत का निर्माण”
Satyanarayan Gangaram Pitroda
“Invent a New India Using Knowledge”

Act Number 22 of 2005
The Indian Parliament
RIGHT TO INFORMATION

Made Available By
Public.Resource.Org

“ज्ञान एक ऐसा खजाना है जो कभी चुराया नहीं जा सकता है”
Bhartṛhari—Nītiśatakam
“Knowledge is such a treasure which cannot be stolen”

BLANK PAGE

BRUTUM FULMEN
IGNORANTIA JURIS NON EXCUSAT
PROTECTED BY COPYRIGHT

IS : 1548 - 1981
(Reaffirmed 1996)

Indian Standard
MANUAL ON BASIC PRINCIPLES OF LOT SAMPLING
(Second Revision)

Second Reprint DECEMBER 2002

UDC 658.562.012.7 : 519.243

© *Copyright* 1982

BUREAU OF INDIAN STANDARDS
MANAK BHAVAN, 9 BAHADUR SHAH ZAFAR MARG
NEW DELHI 110002

Gr 11 *September* 1982

Indian Standard

MANUAL ON BASIC PRINCIPLES OF LOT SAMPLING

(Second Revision)

Quality Control and Industrial Statistics Sectional Committee, EC 3

Chairman	*Representing*
DR P. K. BOSE	University of Calcutta, Calcutta
Members	
SHRI B. ANANTH KRISHNANAND	National Productivity Council, New Delhi
SHRI R. S. GUPTA (*Alternate*)	
SHRI M. G. BHADE	Tata Iron and Steel Co Ltd, Jamshedpur
SHRI C. A. CRASTO	The Cotton Textiles Export Promotion Council, Bombay
DIRECTOR	Indian Agricultural Statistics Research Institute (ICAR), New Delhi
SHRI S. S. PILLAI (*Alternate*)	
SHRI D. DUTTA	The Indian Tube Co Ltd, Jamshedpur
SHRI O. N. AGARWAL (*Alternate*)	
SHRI Y. GHOUSE KHAN	NGEF Ltd, Bangalore
SHRI C. RAJANNA (*Alternate*)	
SHRI S. K. GUPTA	Central Statistical Organization, New Delhi
SHRI A. LAHIRI	Indian Jute Industries' Research Association, Calcutta
SHRI U. DUTTA (*Alternate*)	
SHRI S. MONDOL	National Test House, Calcutta
DR S. P. MUKHERJEE	Indian Association for Productivity, Quality and Reliability, Calcutta
SHRI B. HIMATSINGKA (*Alternate*)	
SHRI A. N. NANKANA	Indian Statistical Institute, Calcutta
SHRI T. R. PURI	Army Statistical Organization (Ministry of Defence)
SHRI ARUN SAXENA (*Alternate*)	
SHRI RAMESH SHANKER	Directorate General of Inspection (Ministry of Defence), New Delhi
SHRI N. S. SENGAR (*Alternate*)	

(*Continued on page* 2)

© *Copyright* 1982
BUREAU OF INDIAN STANDARDS
This publication is protected under the *Indian Copyright Act* (XIV of 1957) and reproduction in whole or in part by any means except with written permission of the publisher shall be deemed to be an infrigement of copyright under the said Act.

(Continued from page 1)

Members	*Representing*
SHRI T. V. RATNAM	The South India Textile Research Association, Coimbatore
DR D. RAY	Defence Research and Development Organization (Ministry of Defence), New Delhi
SHRI A. SANJEEVA RAO	Shriram Fibres Limited, Madras
SHRI G. SAMSON	Indian Telephone Industries Ltd, Bangalore
SHRI P. V. RAO (*Alternate*)	
SHRI P. R. SENGUPTA	Tea Board, Calcutta
SHRI N. RAMADURAI (*Alternate*)	
SHRI R. SETHURAMAN	Electronics Trade and Technology Development Corporation Ltd, New Delhi
SHRI N. SRINIVASAN (*Alternate*)	
SHRI SINGARVELU	State Trading Corporation of India Ltd, New Delhi
SHRI S. SUBRAMU	Steel Authority of India Ltd, New Delhi
SHRI Y. K. BHAT, Director (Stat)	Director General, ISI (*Ex-officio Member*)

Secretary

SHRI PARVEEN KUMAR
Assistant Director (Stat), ISI

Process and Product Control Subcommittee, EC 3 : 6

Convener	
DR P. K. BOSE	University of Calcutta; *and* Indian Institute of Social Welfare and Business Management, Calcutta
Members	
SHRI B. ANANTH KRISHNANAND	National Productivity Council, New Delhi
SHRI R. S. GUPTA (*Alternate*)	
SHRI A. K. BISWAS	The Consultative Committee of Plantation Association, Calcutta
DIRECTOR	Indian Agricultural Statistics Research Institute (ICAR), New Delhi
DR M. N. GOPALAN	Indian Institute of Technology, Bombay
DR S. P. MUKHERJEE	Indian Association for Productivity, Quality and Reliability, Calcutta
SHRI B. K. CHOWDHURI (*Alternate*)	
SHRI RAMESH SHANKER	Directorate General of Inspection (Ministry of Defence), New Delhi
SHRI N. S. SENGAR (*Alternate*)	
DR D. RAY	Defence Research and Development Organization (Ministry of Defence), New Delhi
SHRI M. N. SAPRE	Indian Statistical Institute, Calcutta
SHRI S. SUBRAMU	Steel Authority of India Ltd, New Delhi

CONTENTS

		PAGE
0.	FOREWORD	5
1.	SCOPE	7
2.	TERMINOLOGY	7
3.	CLASSIFICATION AND SPECIFICATION OF QUALITY	10
	3.1 Classification of Quality	10
	3.2 Specification and Measures of Quality	11
4.	SAMPLING INSPECTION	13
	4.1 Comparison of Hundred Percent Inspection and Sampling Inspection	13
	4.2 Inspection by Attributes or Variables	14
5.	PRINCIPLES OF SAMPLE SELECTION	15
	5.1 Sampling Error	15
	5.2 Factors Affecting Sampling Error	15
	5.3 Methods of Sample Selection	16
6.	PLANNING SAMPLE SELECTION	18
	6.1 Preliminaries to Sample Selection	18
	6.2 Formation of Lots	19
	6.3 Determination of Sample-Unit (Item) Size	20
	6.4 Determination of Sample Size	22
7.	ESTIMATION OF LOT QUALITY	22
	7.0 General	22
	7.1 Single Value Estimates of Lot Quality	23
	7.2 Single Value Estimates of Lot Dispersion	24
	7.3 Interval Estimates of Lot Quality	24
	7.4 Interval Estimate of Lot Dispersion	27
	7.5 The Effect of Lot Size on Confidence Interval	28
	7.6 Other Uses of Confidence Intervals	28
8.	SAMPLING FOR LOT ACCEPTANCE	29
	8.1 Risks Involved in Sampling Inspection	29
	8.2 Average Outgoing Quality Limit (AOQL)	36
	8.3 Different Types of Sampling Plans	39
	8.4 Average Amount of Inspection	45

PAGE

APPENDIX A GLOSSARY OF SYMBOLS 46

TABLE 1 FACTORS NECESSARY FOR ESTIMATING σ FROM s ... 49

TABLE 2 FACTORS NECESSARY FOR ESTIMATING σ FROM R OR $\overline{R}$ ... 49

TABLE 3 CONFIDENCE LIMITS FOR PROPORTION OF DEFECTIVE IN A LOT (CONFIDENCE COEFFICIENT 95 PERCENT) ... 50

TABLE 4 VALUES FOR DETERMINING CONFIDENCE LIMITS FOR AVERAGE COUNT PER ITEM IN A LOT (CONFIDENCE COEFFICIENT 95 PERCENT) 52

TABLE 5 VALUES OF h' FOR DETERMINING CONFIDENCE LIMITS FOR THE AVERAGE (ARITHMETIC MEAN) OF A MEASURED CHARACTERISTIC IN THE LOT, USING SAMPLE MEAN WITH SAMPLE STANDARD DEVIATION (CONFIDENCE COEFFICIENT 95 PERCENT) 53

TABLE 6 VALUES OF h FOR DETERMINING CONFIDENCE LIMITS FOR THE AVERAGE (ARITHMETIC MEAN) OF A MEASURED CHARACTERISTIC IN THE LOT, USING SAMPLE MEAN AND SAMPLE RANGE (OR MEAN RANGE) (CONFIDENCE COEFFICIENT 95 PERCENT) ... 54

TABLE 7 FACTORS f_1 AND f_2 FOR DETERMINING CONFIDENCE LIMITS FOR VARIABILITY (STANDARD DEVIATION) OF A MEASURED CHARACTERISTIC IN THE LOT, USING SAMPLE STANDARD DEVIATION (CONFIDENCE COEFFICIENT 95 PERCENT) 55

TABLE 8 FACTORS f_3 AND f_4 FOR DETERMINING CONFIDENCE LIMITS FOR VARIABILITY (STANDARD DEVIATION) OF A MEASURED CHARACTERISTIC IN THE LOT, USING SAMPLE RANGE (CONFIDENCE COEFFICIENT 95 PERCENT) 56

AMENDMENT NO. 1 MARCH 1993
TO
IS 1548 : 1981 MANUAL ON BASIC PRINCIPLES OF LOT SAMPLING

(Second Revision)

(*Cover page and all other relevant pages, designation*) — Substitute 'IS 1548 (Part 1) : 1981' *for* 'IS 1548 : 1981'.

(*Cover page, pages* 1 *and* 5, *title*) — Substitute the following for the existing title:

'MANUAL ON BASIC PRINCIPLES OF LOT SAMPLING

PART 1 ITEMIZED LOT SAMPLING'

(Second Revision)

(MSD 3)

Printed at Simco Printing Press, Delhi

Indian Standard

MANUAL ON BASIC PRINCIPLES OF LOT SAMPLING

(Second Revision)

0. FOREWORD

0.1 This Indian Standard (Second Revision) was adopted by the Indian Standards Institution on 30 June 1981, after the draft finalized by the Quality Control and Industrial Statistics Sectional Committee had been approved by the Executive Committee.

0.2 From the early days of industrial production, producers have tried to maintain uniformity in man, machine, raw materials and other factors influencing the process of manufacture with the hope of turning out products of uniform quality. But none of these is infallible and cases of irregularity creep into the production processes inadvertently. Controlling the quality of products, so as to maintain it at a given level and within specification limits, is a major problem with all producers. To assist the producers in tackling such problems in a scientific manner, IS : 397 (Part I)-1972* and IS : 397 (Part II)-1975† were published, which outline the control chart method of controlling quality during production.

0.3 Similarly, inspection of lots of products, so as to get assurances about their quality, is a vital problem with the consumers. With phenomenal increase in the manufacturing capacity due to the machine-age, inspection of each and every unit of production becomes extremely uneconomical. In many cases, like those involving destructive or costly testing, inspection of each and every unit is not to be conceived at all. The only alternative method available in these cases is sampling inspection which consists in selecting a smaller number of units, called a sample, from the lot or the production and then deciding about the quality of the latter on the basis of the results obtained from the inspection of the sample units. The sampling inspection is generally more practical, quick and economical.

0.4 The standard was originally issued in 1960 and revised in 1969, with a view to acquainting the producers and consumers with the concepts and

---

*Method for statistical quality control during production: Part I Control charts for variables (*first revision*).

†Method for statistical quality control during production: Part II Control charts for attributes and count of defects (*first revision*).

principles of sampling inspection. In view of the experience gained with the use of the standard in course of years, it was felt necessary to revise it again so as to make the concepts more up-to-date and also to include some more useful material. It is hoped that this manual will continue to contribute towards the increased use of sampling plans by consumers for judging the acceptability of their purchases and by manufacturers for purposes of improving the quality of their products.

0.4.1 Besides, this manual is also expected to be of assistance to various committees of ISI in familiarizing them with the concepts and principles of sampling and evolving sampling plans suited to individual specifications.

0.5 In this manual, refinements in statistical theory and advanced sampling techniques have been avoided and the principles of sampling have been simplified, so as to make them practicable to a great extent. Particular emphasis is laid on the interpretation of the data resulting from sampling in the problem of estimation of lot quality and lot acceptance, although many of the concepts and definitions, principles of sample selection, etc, are equally applicable to problems connected with process control, industrial design of experiments and so on.

0.6 This manual is one of a series of Indian Standards pertaining to techniques of statistical quality control. Other standards in the series are given on Page 57.

0.7 In preparing this manual, considerable assistance has been derived from a draft manual compiled by Dr A. Matthai of the Indian Statistical Institute and also from the following publications:

PEARSON (E S) and HARTLEY (H O), Ed. Biometrika tables for statisticians. Vol I, Ed 2. Cambridge University Press, London.

RAO (C R), MITRA (S K) and MATTHAI (A). Formulae and tables for statistical work. Statistical Publishing Society, Calcutta.

Sampling plans in ASTM specifications — A report of a preliminary survey conducted by task group 41 of ASTM Committee E-11 on quality control of materials.

JACKSON (J E) and ROSS (E L). Extended tables for use with the 'G' test for means. Journal of the American Statistical Association. Vol 50, 1955.

DODGE (H F) and ROMIG (H G). Sampling inspection tables. 1959. John Wiley & Sons Inc, New York.

0.8 In reporting the results of a test or analysis, if the final value, observed or calculated, is to be rounded off, it shall be done in accordance with IS : 2-1960*.

---

*Rules for rounding off numerical values (*revised*).

1. SCOPE

1.1 This manual explains the various statistical concepts underlying sampling inspection and lays down some of the basic principles of sampling lot by lot.

1.2 This manual deals with sampling inspection techniques as a means to solve two types of problems, namely, (a) estimation of lot quality, and (b) lot acceptance.

1.3 Specific procedures necessary for sampling a particular material giving details of sampling implements, methods of preparing specimens, etc, are not covered in this manual.

2. TERMINOLOGY

2.0 For the purpose of this manual, the following definitions shall apply. The symbols used in this standard, including those for some of the terms defined below are explained in Appendix A.

2.1 Hundred Percent Inspection (or Screening) — Inspection in which all the ' items ' in the ' lot ' are inspected.

2.2 Sampling Inspection — Inspection in which only a portion of the ' lot ' is inspected.

2.3 Item — Ultimate unit of product or material on which inspection will be performed.

In most of the cases, an item consists of a single discrete article which can be defined without difficulty; but, in the case of materials which take continuous form, such as cables or which are available in bulk, such as iron ore, no separate or discrete units are ordinarily provided and in such cases it would be necessary to give an unambiguous definition of the ultimate inspection unit.

2.3.1 *Discrete Item* — The single integral unit of product which when physically partitioned (broken down, disassembled, etc) cannot be used as originally intended, for example, bolt, nut, nail, brick, etc.

Sometimes, a discrete item may be a more complicated unit, built up of different smaller units or parts each with different properties, for example, hinge, shoe, electrical lamp, ball bearing, etc.

2.3.2 *Item for Continuous or Bulk Material*

2.3.2.1 When a material is extruded, drawn, rolled or woven and assumes a continuous form, it may be conveniently possible to define an item as a specimen of specified length (as in the case of tubes, wires or cloth) or as a specimen of specified area (as in glass or metal sheets).

2.3.2.2 When the material (solid, liquid or gas) is available in bulk form (for instance, coal, iron ore, sand, asphalt, paint, chlorine or ammonia), the item is defined as a relatively small portion of specified weight or volume of the bulk. Such an item defined for the bulk may consist of one or more of what are sometimes called increments.

2.3.2.3 Items defined for continuous and bulk materials would be similar to discrete items and treated similarly in many respects. There is, however, one main difference that such items may be combined together or further partitioned, broken down, etc, physically without altering the characteristics of the material. In view of this, the determination of the size of the item, which should not be arbitrary, becomes important in the case of continuous and bulk materials.

2.4 Package — Usually a rational or homogeneous unit containing one or more items as defined in **2.3**.

Materials and products are sometimes put into packages at the time of collection from the source or at the manufacturing plant, for example, bale of wool, tin of paint, bag of cement, box of nails, etc. A package could itself be an item under certain circumstances (*see* **5.3.5**).

2.5 Lot — A batch, a group, a continuous stream or a bulk of product or raw material submitted for inspection.

A lot may be a shipment, a consignment, a delivery or a portion of them. A lot may be a rational group, such as material from one heat or batch, or may be a mixture of several of these. A lot is viewed as an aggregate of items.

2.5.1 *Item-Lot* — An aggregate of ' Discrete Items ' or an aggregate of ' Items ' of continuous or bulk material which have been rendered discrete in the form of specimens or increments, for example, batch of screws turned out by a machine, steel rails produced in a day, pieces of 6-metre length of rayon, consignment of 5-litre tins of lacquer, etc.

2.5.2 *Bulk-Lot* — A lot not characterized by a similar ' Discrete Items ' but presented in the form of continuous or bulk material, for example, coal or iron ores in a ship's hold(s), sand or lime in rail wagon(s), roll(s) of cables, etc.

2.5.3 *Homogeneous Lot* — A lot consisting of product or material turned out by a process in state of control.

Homogeneity of a lot is to be understood as uniformity of product or material within the lot. Products and material from even a controlled process will not be identical or perfectly uniform. Thus, perfectly homogeneous lots rarely occur in practice. Different lots may differ in degree of homogeneity.

2.5.4 *Lot-Size* — Number of ' Items ' in a ' Lot '.

2.5.5 *Sub-lot* — Sub-division of a ' Lot '.

For considerations of economy in inspection or efficiency of sampling, a lot may be divided into sub-lots, if possible. This division into sub-lots may be carried on to two or more stages as well; but, inference is normally made about the lot as a whole and not about each sub-lot.

2.6 Sample-Unit — Unit in terms of which the selection is made.

A sample-unit is generally an ' Item '. In some situations, a ' Package' or a ' Sub-lot ' may be a sample-unit. In the case of bulk material, the unit in terms of which the selection is made is sometimes called an increment.

2.7 Sample — Collection of all ' Sample-units ' (generally items) selected from a ' Lot '.

Usually the sample would contain a number of ' Sample-units ' (two or more). Sometimes, the word sample is used to refer to ' Sample-unit '; this is not recommended unless the sample consists of one ' Sample-unit ' only.

2.7.1 *Gross Sample* — The aggregate of all ' Items ' (increments) selected from a single bulk-lot or a sub-lot.

2.7.2 *Sample Size* — The number of ' Sample-units ' in the ' Sample' (generally the number of ' Items ' in the ' Sample ').

2.8 Sampling Error — Error due to sampling in the inference about a ' Lot '.

The manner in which a sampling error is constituted will differ from problem to problem. Thus, in the problem of estimating lot quality, sampling error will be the deviation of the sample estimate from the actual value of lot quality; in the problem of lot acceptance, accepting a bad ' Lot ' or rejecting a good ' Lot ' on the basis of a ' Sample ' may be considered as error due to sampling.

2.9 Sampling Efficiency — The efficiency of a sampling procedure as judged from the ' Sampling Error ' involved, whatever be the way in which the ' Sampling Error ' is defined and measured.

The smaller the ' Sampling Error ', the more efficient the sampling will be. Efficiency of sampling, so conceived, does not take into account the cost of sampling.

2.10 Sampling Plan — A statement of the sampling procedure and the rule for making inferences about the ' Lot '.

It will provide the definitions of lot, sample unit, sample size, method of sample selection and criteria for inferences, so as to satisfy specific stipulations.

2.11 Quality — Any characteristic, that is of interest, of a product or material. A product or material may be described, specified or inspected in terms of one or more quality characteristics.

2.12 Defective Item — An 'Item', the quality of which does not meet specified requirements.

2.13 Defect — Failure to meet the requirement imposed on an 'Item' with respect to a single characteristic.

The defects appearing in each 'Item' (for one or more characteristics) are usually counted. Thus, there may be none, one or more defects in an 'Item'. Defects are sometimes classified into critical defects, major defects and minor defects, depending on the extent to which they may cause damage. (When the number of defects in an 'Item' exceeds a specified limit, the 'Item' turns out to be defective).

2.14 Observation — The value of a characteristic determined as a result of an inspection or test.

2.15 Sample Average ($\bar{x}$) — Sum of the observations divided by the number of items in the sample.

2.16 Lot Average (μ) — Sum of the observations on all items divided by the total number of items in the lot.

2.17 Sample Standard Deviation (s) — The square root of the quotient obtained by dividing the sum of squares of deviations of the observations from their mean by one less than the number of observations in the sample.

2.18 Lot Standard Deviation (σ) — The square root of the mean of the squares of the deviations of all observations in a lot from their mean.

2.19 Range (R) — The difference between the largest and the smallest observations or test results in a sample.

2.20 Coefficient of Variation (v) — The absolute value of the ratio of the standard deviation to the average. Sometimes, the coefficient of variation is also expressed as a percentage by multiplying this ratio by 1C0.

3. CLASSIFICATION AND SPECIFICATION OF QUALITY

3.1 Classification of Quality — Quality characteristics are broadly classified into the following three types, according to the method of inspection:

a) *Attributes*, according to which each item inspected is classified into either of the two categories depending upon presence or absence of the characteristic(s):

Examples:

Attributes	*Category* 1	*Category* 2
i) Colour of sugar	White	Not white
ii) Existence of blow-holes	With holes	Without holes
iii) Conformity of bolt diameter to the specification	Conforming	Not conforming
iv) Grain size	Passing through the sieve of specified size	Not passing through the sieve of specified size

b) *Count of defects*, according to which the number of defects on each inspected item is counted, such as the number of blisters/bubbles on test tubes, number of end-breaks per unit length of yarn, etc.

c) *Variables*, according to which each item inspected gives rise to a measurement of a characteristic on a continuous scale, such as the tensile strength of steel wires, specific gravity of paint, etc.

3.1.1 Variables type and count of defects type of data can be converted to attributes type of data with the help of any specification on the measurable characteristic or limit on the number of defects. Thus, a bolt of diameter 11·5 mm would be classified as ' good ' if the specification stipulates a minimum diameter of 11·0 mm or a copper sheet having no defect would be classified as ' good ' if the specification stipulates a maximum of one defect. However, it may be noted that any such conversion of the variables or count of defects type of data to the attributes type results in a certain loss of information.

3.2 Specification and Measures of Quality

3.2.1 Whether it is a lot or a sample that is subjected to inspection, ultimately the inspection is performed on the item. Thus, the lot quality and sample quality could be regarded as built out of item quality.

3.2.2 *Item Quality* — When item quality is of the attributes type, it is specified by means of a standard specimen or in terms of specification limits or as a result of some tests. For example, a phial of sugar with certain whiteness for inspecting colour of sugar, or a pair of ' Go ', ' No-Go ' gauges corresponding to tolerances for the diameter of bolt or the presence of negative or positive reaction for testing the genuineness of honey by Fiehe's test or passing or failing of heating elements in high voltage test.

3.2.2.1 When the item quality is of the count of defects type, only an upper limit is specified. For example, the number of dead pin knots per face of plywood panel should not be more than 4.

3.2.2.2 When item quality is of the variables type, it is specified in terms of limit values. Depending on the quality characteristic, there will ordinarily be either only one (lower or upper) or two (lower and upper) specification limits. For example, it may be specified that the moisture and volatile matter in shaving soaps should not be more than 12 percent by mass, the elongation percent of aluminium conductors should not be less than 25; nicotine content in cigarettes should be between 1 and 3·5 percent.

3.2.3 *Lot Quality* — Whichever be the type of item quality (attributes, count of defects or variables), the corresponding lot quality can be described in terms of the percentage (or fraction or number) or defective items in the lot.

3.2.3.1 When the item quality is of the attributes type, the only measure of lot quality is the percentage defective.

3.2.3.2 When the item quality is of the count of defects type, the lot quality can also be expressed in terms of number of defects per item or for 100 items.

3.2.3.3 When the item quality is a variable and measured, the lot quality can also be expressed in terms of either one or both of the following:

a) The average value of the variable characteristic among the items in the lot (*arithmetic mean* is a common average).

b) The variability of the characteristic among the items in the lot (*standard deviation* is a common measure of variability).

3.2.4 *Sample Quality* — So far as a sample, like a lot, contains more than one item, sample quality will be judged from the quality manifestations of the different items constituting the sample. Corresponding to each of the lot quality measures described in **3.2.3**, there will be a sample quality measure. Thus, for attributes type of item quality, a sample percentage defective is invariably used. For count of defects type of item quality, sample quality would be the average count of defects of all the items in the sample. For variables type of item quality, sample quality would be either one or both of : (a) average quality of the items in the sample (arithmetic mean), and (b) variability of the quality of the items in the sample (standard deviation; the range is also used as a simpler substitute for standard deviation in the case of sample quality). The sample percentage defective is also a possible measure of lot quality when item quality is a count of defects or measurement, but is not efficient.

3.2.4.1 It should be noticed that lot quality and sample quality are always expressed quantitatively, while item quality can be either qualitative or quantitative.

4. SAMPLING INSPECTION

4.1 Comparison of Hundred Percent Inspection and Sampling Inspection

4.1.1 The merit of sampling consists in the fact that inspection of a sample of relatively small size taken from a lot would enable one, with considerable economy and accuracy, to make an inference about the lot. A brief comparison of the relative advantages and disadvantages of hundred percent inspection and sampling inspection may be enumerated as follows:

	Hundred Percent Inspection	*Sampling Inspection*
a)	Total cost of inspection per lot is high, sometimes prohibitive, if the test or analysis involved is costly.	Total cost of inspection per lot is low. Hence, more characteristics on each item could be studied.
b)	Quite time consuming, if the lot size is large.	Decisions can be arrived at quickly.
c)	Generally not feasible due to limitations of skilled personnel, complex instrumentation, etc.	Generally feasible.
d)	More handling damage during inspection.	Less handling damage during inspection.
e)	Complete accuracy of inference is seldom attained due to inspection errors arising out of fatigue, negligence, difficulty of supervision, etc.	Inspection can be organized and controlled more efficiently. Reasonable accuracy of inference is possible. Also measure of sampling error can be obtained.
f)	Not feasible when the inspection involves destructive tests (for example, testing life of wireless tubes, breaking strength, combustibility of material, etc).	Only method possible when the inspection involves destructive tests.
g)	The supplier has little incentive to improve his products, since only defective items are rejected and sent back to him for replacement.	Because of the small (known) percentage of defectives that has to be permitted, the producer has incentive to improve the quality of his supplies.
h)	The method available when no risk or error due to sampling can be taken (such as inspection of critical parts, safety devices, etc).	Some risk (measurable), due to sampling error, will have to be tolerated.

4.1.2 On the whole it would be seen that sampling inspection has many more advantages and has wider practical applications in industry as compared to hundred percent inspection. However, when hundred percent inspection becomes inevitable (in cases where incoming quality is either inferior to the desired quality or less than the ratio of cost of inspection per item to the cost of defective item being accepted), it is advisable to superimpose a sampling inspection, so as to check the efficacy of hundred percent inspection.

4.2 Inspection by Attributes or Variables

4.2.1 When the items are inspected for their quality and results are expressed as attributes, the procedure is called 'Sampling Inspection by Attributes'. For further details, *see* IS : 2500 (Part I)-1973[*].

4.2.2 When the items are inspected for their quality and results are expressed as count of defects, the procedure is called 'Sampling Inspection by Count of Defects'. For further details, *see* IS : 2500 (Part I)-1973[*].

4.2.3 When items are inspected for their quality and results are expressed in terms of units of measurement on a continuous scale, the procedure is called 'Sampling Inspection by Variables'. For further details, *see* IS : 2500 (Part II)-1965[†].

4.2.4 *Comparison of Inspection by Attributes and by Variables* — When a choice between attributes inspection and variables inspection is relevant, as when a characteristic can either be measured or gauged, the following are some of the important considerations providing a basis for the appropriate choice [*see also* **3** of IS : 2500 (Part II)-1965[†]]:

	Attributes Inspection	*Variables Inspection*
a)	Since the inspection is performed either visually or by gauging, the cost of inspection per item is low.	Since the inspection is done by measurement and the observations may also have to be recorded, it involves more time and labour The cost of inspection per item is, therefore, more.
b)	For the same degree of efficiency in inference made about the lot, more items need be inspected.	For the same degree of efficiency in inference made about the lot, less items need be inspected.

---

[*] Sampling inspection tables: Part I Inspection by attributes and by count of defects (*first revision*).

[†] Sampling inspection tables: Part II Inspection by variables for percent defective.

c) Computation of sample quality from item quality is less complicated.	Computation of sample quality from item quality is more complicated.
d) Inspection may be slightly subjective, especially in respect of visual characteristics.	Inspection is more objective and minimizes the possibilities of misclassification.
e) More broad based and has wider applications.	Based on assumption that the characteristic under consideration has a normal distribution.
f) More than one characteristic can be considered at a time, finally resulting in classification of the item as defective or non-defective, for example, diameter, length or thread of a bolt may be considered in classifying an item as defective or non-defective.	Only one characteristic can be taken into account at a time.

5. PRINCIPLES OF SAMPLE SELECTION

5.1 Sampling Error — The logic of sampling is that of generalizing from a portion about the whole; on the basis of evidence gathered from a sample drawn from a lot, an inference is made about the lot. Sampling then is a short-cut method for investigation of lots and naturally offers considerable economy in time and labour. Against this saving, however, is the error characteristic of all generalizations. The nature of this error would be different for different problems. Since a sample constitute only a part of the lot, many samples can be drawn from a lot. These samples may differ from one another with regard to the inference about the lot. Such differences constitute sampling errors.

5.2 Factors Affecting Sampling Error — If the lot consists of identical items there will not be any sampling error whatever be the size of sample, and obviously a sample consisting of a single item would be the most economic choice. Further, if the sample exhausts the entire lot, there is no question of sampling error even if there is variation from part to part. When, however, either of these is not the case, the sampling error will depend on the degree of homogeneity of the lot as well as the relative size of the sample. The more the homogeneity in the lot or larger the size of the sample, the smaller would be the sampling error. The method of sample selection also contributes to sampling error.

5.3 Methods of Sample Selection

5.3.1 *Bias in the Selection of Sample Units* — Bias in the selection of sample units can considerably affect the sampling efficiency. Such bias in sampling can generally arise in the following ways:

a) Certain positions in the lot may be given preference, with the result that bias in sampling will be introduced if the position of items in the lot is not independent of the item quality. For example, good articles may be more at the top in a container; machined parts coming out of lathe, at different intervals of time, may differ in dimensions due to tool-wear; and larger particles may be at the bottom in a conical heap of sand dropped from a chute.

b) Because of the existence of variability in item quality, a selection procedure may be such as to give preference for certain item qualities. For example, an inspector may be able to spot defectives quickly and may be inclined either to collect more of them or avoid them in the sample.

c) Certain short cut methods for collection of sample or ignorance on the part of the sampler may also lead to a biased sample. For example, in sampling loaded wagons of iron ore, the samples collected from the top surface only may render the sample unrepresentative, or in picking articles from the container more of the bigger ones which have a higher value of the characteristic under consideration may be selected.

A method of sample selection which avoids bias is random sampling, which is basically assumed in the theory of sampling for the ultimate selection of the sampling unit.

5.3.2 *Random Sampling* — A random sample may be defined as the sample drawn in such a manner that the chance for inclusion of any item in the sample is predetermined. Such a chance is independent of the quality of the item and is also independent of the quality of other items selected for the sample. When the chance for inclusion of any item in the sample is same, it is referred to as simple random sample. The procedure to collect a proper simple random sample is to use some mechanical device or its equivalent which will eliminate bias in selection. For instance, the items in the lot might be serially numbered and a device like the wheel of chance used or each number be represented on different cards which are shuffled well and the requisite number of cards selected blindly, thus providing the serial numbers of items which form the sample. Alternatively, a table of random numbers, for example, the one given in IS : 4905-1968* may be used. Practicable schemes of any randomizing procedure may be devised to suit the configuration of the lot, other physical circumstances, the item quality, the inspection operation, etc.

---

*Methods for random sampling.

5.3.2.1 *Approximation to random selection* — From the practical point of view, it may be difficult to arrange for ideal randomization in many cases. With respect to certain bulk lots, such as those composed of materials like liquids or powders, it may be possible to stir or mix the lot thoroughly and then the requisite number of sample units may be selected blindly to obtain a sample as good as a random sample. In other situations, the following general rule of selection is recommended which results in a close approximation to random selection.

Select sample units blindly from all parts of the lot. For this, the lot should be presented in such a way that the inspector has access to all parts of the lot. The position from which units are selected within any part should also be varied from part to part.

5.3.3 *Systematic Sampling* — When the items are presented in an orderly manner, it is quite convenient to select sample unit systematically at regular intervals of position or time. Initially, one item is chosen from the lot at random and, thereafter, items are selected regularly at pre-determined intervals till the requisite number of sample items are collected. In practice, this method has been found to be quite a good approximation to random sampling described in **5.3.2**.

5.3.4 *Stratified Sampling* — When a lot consists of items which can be divided into a certain number of more homogeneous groups or sub-lots or strata, the method of stratified sampling becomes the obvious choice. According to this method, each sub-lot or stratum is sampled separately, so as to obtain a sample representative of the entire lot. It has been found that the sample obtained by this method of sampling is generally more efficient than a simple random sample (of the same size) drawn from the entire lot in the sense that sampling error will be less in the former case. The more the homogeneity of the sub-lots, the more efficient would be the plan. When the sub-lots are of the same size, the proportional allocation of sample size to different sub-lots becomes very simple. Even though there is no prior information for dividing a lot into sub-lots on the basis of the homogeneity of the sub-lots, this sub-division is adopted for assisting in the collection of a representative sample from the lot. It is precisely for this reason that sub-division principle is being widely followed in the sampling of all ores and raw materials.

5.3.5 *Sampling in Stages* — There are situations in which the lot is presented in a large number of sub-lots or packages in which neither a random sample of items from the entire lot nor a proportional sample of items from each and every sub-lot or package may be feasible. For example, in the case of a lot containing packages it would often mean the opening of all the packages or, alternatively, mixing the items in the packages together before selecting the items. The cost of locating, opening and sampling of packages is generally many times that of inspecting an individual item, making it necessary that the maximum number of items from the minimum number of packages should be inspected, without, of course, losing sight of the sampling efficiency. In such cases, instead of sampling from all the packages, a random sample of packages is first chosen and from each package selected a random sample of items proportional to size of the package is chosen. This procedure may be extended to more than one stage. For example, high permeability nickel iron is delivered in rolls weighing several thousand kilograms. A primary sample-unit will be a heat, a secondary sample-unit will be a roll and a tertiary sample-unit will be a specimen (item) from within a roll. Again in the case of coal, a primary sample-unit may be the mine; a secondary sample-unit, a truck-load; and a tertiary sample-unit, an increment from the truck-load.

5.3.6 Different methods of sample-unit selection have been suggested in the foregoing section to suit different situations. For a more exhaustive discussion on the relative merits and demerits of all these methods as also some others, *see* IS : 4905-1968*. However, in this manual, for purposes of making inference dealt with in **7** and **8**, no distinction will be made between samples on account of differences in method of selection of sample-units; samples of the same size, whatever be the way in which the units in it have been selected, will be considered similar and equivalent to the simple random sample that can be obtained from the lot.

6. PLANNING SAMPLE SELECTION

6.1 Preliminaries to Sample Selection — In order that the assumptions in sampling theory are not invalidated and that the maximum possible sampling efficiency is achieved, proper planning of sample selection is necessary. For planning purposes, as much information about the lot (such as the nature of items and the quality characteristics) as available prior to inspection of the items will be useful. The important preliminaries to sample selection are:

a) Statement of the type of inference or decision desired, whether assessing lot quality or lot acceptance;

b) Defining as to what forms the lot and the size of the lot (*see* **6.2**);

c) Choice of the sample unit, including determination of its size in the case of bulk material (*see* **6.3**);

d) The size (the number of items) of the sample to be taken from the lot (*see* **6.4**);

---

*Methods for random sampling.

e) Method of sample selection (*see* **5.3**);

f) Choice of item quality (specifying a non-defective item, if relevant);

g) Choice of lot quality (specifying an acceptable lot, if relevant); and

h) Criteria to be used for making the inference (*see* **7** and **8**).

The next step would be to provide a sampling plan to suit the purpose in view. In order to provide a sampling plan, some stipulations as regards the permissible limits of errors in inference due to sampling will be necessary. The nature of stipulations will be different for different types of inference, whether assessing lot quality or lot acceptance.

6.2 Formation of Lots

6.2.1 Formation of lots will have to be planned with respect to the following:

a) Type of lot (stationary lot or moving lot, item-lot or bulk-lot);

b) Size of the lot;

c) Homogeneity of the lot, and of sub-lots; and

d) Identity of lots and accessibility to items in the lot.

6.2.2 *Stationary and Moving Lots* — From a stationary lot, in which the items are presented simultaneously, all the sample items may be obtained at one time. In a moving lot, as the lot flows past the point of inspection one or a few items at a time are selected. In many situations stationary lots offer advantages over moving lots, since their identification and disposition are easy and they admit re-sampling and sequential sampling. However, they create stacking-space problem, especially when the items within are to be made easily accessible. But in sampling bulk materials, like coal, grains, molten metal, etc, it is often more convenient to sample while the material is in motion.

6.2.3 *Size of Lot* — The general rule in the formation of lots is to make the lot size as large as possible provided that a reasonable degree of homogeneity is maintained. This is recommended because, for any given degree of efficiency of sampling and for the same degree of homogeneity in the lot, the size required for the sample will not increase as rapidly as the lot size and will not increase after a certain size for a lot (*see also* **7.5** and **8.1.3**). Thus, a sample of 300 items from a lot of 10 000 items will be more efficient than a sample of 100 items from a lot of 1 000 and will be almost as efficient as a sample of 300 items from a lot of 100 000 items. This means that the cost of inspection per item submitted for sampling inspection will be much less if the lot handled is larger in size. But the size of the lot will have to be limited on account of the following factors:

a) The formation of larger lots may result in the inclusion of items differing more widely in quality;

b) The production or supply of material may be such that the accumulation of large lots will be over a long period;

c) Due to storage and handling difficulties, very large lots will not be feasible; and

d) The economic consequence of rejection of larger lots because of the cost of scrapping, the cost of detailed inspecting or the cost of reworking them.

In forming a lot, therefore, a compromise will have to be made in respect of its size.

6.2.4 *Homogeneity of Lots* — As the efficiency of sampling depends on the degree of homogeneity of the lot, efforts should be made not to have a mixed lot as far as possible and to confine the lot to material or products originating from essentially similar conditions, such as raw materials or components of the same source, products manufactured by the same production or assembly line or moulds or patterns, items produced from a single batch or from one setting of the machines during the same day or shift, etc. Product or material coming out of a process under statistical control forms homogeneous lots [*see* IS : 397 (Part I)-1972* and IS : 397 (Part II)-1975†]. When, however, sub-lots from different sources are combined to form a lot, as far as possible, the identity of the sub-lots should be preserved, so that efficient stratification can be achieved.

6.2.5 *Accessibility of Item in the Lot* — It is also important from the point of view of convenience of inspection that the lot is easily identifiable and is in the form that the inspector will have easy access to all parts of the lot to select the representative sample.

6.3 Determination of Sample-Unit (Item) Size

6.3.1 While the item, which is the ultimate sample-unit is clearly defined in item-lots, the determination of the item size (such as the weight or volume of the increment, or the length or area of a specimen) in bulk-lots presents certain problems. In the case of bulk-lots, usually larger the size (area, volume or weight) of the item, greater would be the sampling

---

*Method for statistical quality control during production: Part I Control charts for variables (*first revision*).

†Method for statistical quality control during production: Part II Control charts for attributes and count of defects (*first revision*).

efficiency; but beyond a certain size of the item, the rate of gain in sampling efficiency may be negligible. Besides, from practical considerations, the large size of the increment for bulk products like iron ore, coal, etc, may also pose problems like limitations of manual sampling, introduction of errors due to crushing, etc. However, if sampling efficiency is the only criterion for determining the item size, then that size, beyond which the rate of gain in efficiency would be very small, may be taken as the appropriate item size.

6.3.1.1 For determining the item size following this principle, statistically designed experiments should be conducted covering different sizes for the item with respect to the item quality characteristic of interest. Corresponding to each size considered from the values of the item quality, a measure of variability in item quality should be calculated. These values plotted against different sizes for the item would give a curve. In general, this curve would slope downwards in the direction of increase in the item size and would be more or less flat after a certain point. The size corresponding to this point would be an appropriate item size.

6.3.1.2 In an investigation conducted on hard coke, large (nominal size 150 to 38 mm), 7 series of 50 increments each were collected. The increments in each series were of approximately equal size which were analysed by qualified chemists for their ash contents individually. The results of the analyses were obtained over a period of time. The average size of the increment in each series and the corresponding measure of variability (coefficient of variation, *v*) of ash content were as follows:

Series	*Average Size of Increment (Weight in kg)*	*Coefficient of Variation, v, of Ash Content*
1	1·7	3·8
2	2·0	3·5
3	2·6	3·1
4	4·0	2·5
5	4·8	2·3
6	5·2	2·2
7	6·0	2·1

The coefficients of variation of ash content between the increments have been plotted against the average size of the increments in Fig. 1.

On the basis of this investigation, about 5 kg could be taken as a suitable size of the increment, as increments of larger size do not decrease the coefficients of variation appreciably.

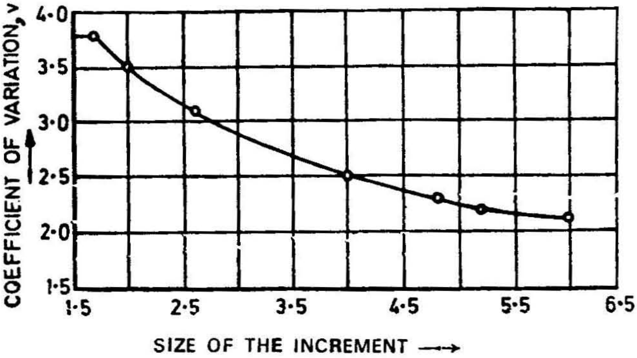


FIG. 1 DETERMINATION OF INCREMENT SIZE

6.4 Determination of Sample Size — It is not possible to have a single rule for determination of the size of the sample. The number of items to be in the sample is dependent on the extent of error due to sampling that can be tolerated which, in turn, depends to a large extent on the degree of homogeneity of the lot. The size of the lot also comes into picture more from practical considerations than any statistical reasoning. Of course, it can be stated that the effect of lot size on sampling efficiency is greater when the lot size is small. But in most industrial usages, where large lot sizes are usually encountered, it is the actual size of the sample that is more important than what percentage sample size forms of the lot. In view of this, specifying the sample as the percentage of the lot size, is not recommended (*see* **7.5.1**, **8.1.2** and **8.1.3**).

For many purposes sample size may have to be large. The conventional assumption that even a small sample could be representative of the lot is to be treated with reserve. The sample size should be determined with due regard to the type of inference, the homogeneity within the lot, size of the lot and with a view to minimizing the cost of inspection. Detailed recommendations for arriving at the desired sample size for estimation of lot quality* or for lot acceptance† are covered in separate Indian Standards.

7. ESTIMATION OF LOT QUALITY

7.0 General — Of the two main problems, namely, estimation of lot quality and lot acceptance dealt with in this manual, the first is described in **7.1**

**See* IS : 5002-1969 Methods for determination of sample size to estimate the average quality of a lot or process.

†*See* IS : 2500 (Part I)-1973 Sampling inspection tables: Part I Inspection by attributes and by count of defects (*first revision*).

to 7.5. While there is much in common with the two problems with regard to the methods for sample selection and the sampling theory used, a sampling plan devised for one may not be quite adequate or suitable for the other since the requirements of each problem would be different.

7.0.1 In estimating lot quality on the basis of a sample, the sample quality consisting of a single value will be adequate for certain purposes. Thus, the sample percentage defective would provide an estimate of the percentage defective in the lot, or the sample arithmetic mean would estimate the arithmetic mean in the lot. But, in view of the error due to sampling, it is often desirable to give an indication of the reliability of the estimate. This is done by providing two limits which are likely to include the true value of lot quality. These two limits, called the upper and the lower limits determine an interval called the confidence interval. This will have a confidence coefficient associated with it. An interval with a confidence coefficient of 95 percent would signify that if, following a given rule, confidence intervals are constructed based on samples of a given size repeatedly drawn from the same lot, then on an average 95 percent of these intervals will contain the true value of lot quality. It is in this sense that it will be ascertained that an interval constructed on the basis of a sample will contain true lot quality. Though confidence intervals for any confidence coefficient like 90 percent, 99 percent, etc, could be computed, the tables given in this manual pertain to a confidence coefficient of 95 percent since it is more widely used in practice.

7.1 Single Value Estimates of Lot Quality

7.1.1 *Lot Fraction Defective* — The sample fraction defective (p) which is obtained by dividing the number of defectives found in the sample by the sample size gives a good estimate of the lot fraction defective (p'). In many situations, this fraction is also expressed as a percentage by multiplying it by 100.

7.1.2 *Average Count of Defects per Item in the Lot* — The average count of defects per item in the sample ($\bar{c}$) which is obtained by dividing the total number of all the defects found in the sample by the sample size, gives a good estimate of the average count of defects per item in the lot ($\bar{c}'$). In many situations, this average is also expressed as count of defects per 100 items by multiplying it by 100.

7.1.3 *Lot Average* (μ) — When the characteristic under consideration is of the variables type, the sample average ($\bar{x}$), which is obtained by dividing the sum of all the observations by the sample size, is a good estimate of the lot average.

7.2 Single Value Estimates of Lot Dispersion

7.2.1 *Lot Standard Deviation* (σ) — From the different observations on the sample items, the sample standard deviation (s) would be calculated as the square root of the quotient obtained by dividing the sum of squares of deviations of the observations from their average by one less than the sample size. When the sample size is small, say less than 20, the value of the sample standard deviation (s) is to be divided by a suitable factor c_1 given in Table 1 (*see* P 49) for obtaining satisfactory estimate of the population standard deviation (σ). However, for samples of sizes 21 and above, the sample standard deviation itself would give an estimate of the population standard deviation.

7.2.1.1 If the computation of sample standard deviation (s) from the sample is considered difficult for any reason, the range (R) which is not as efficient as the standard deviation, but simpler to calculate, can be used to estimate σ. The range, however, is a satisfactory substitute for the sample standard deviation only for smaller sample sizes and its use is recommended only for values of n less than, say, 20. For this reason, when range is to be used for large samples, a gross sample of size mn may, for this purpose, be subdivided into m samples each of size n and the average of m ranges (that is, mean range $\bar{R}$) used instead of using a single R based on mn measurements. Sub-sample size close to 8 are recommended for obtaining the mean range. However, in most of the quality control practice, sub-groups of size 5 are normally used for the sake of convenience. Since the range in the lot by itself is not a common or useful measure of lot dispersion, the sample range R or mean range $\bar{R}$ would be used to estimate the lot standard deviation. To render R (or $\bar{R}$) an estimate for σ, it is to be divided by a factor d_2 given in Table 2 (*see* P 49) which depends on the value of n.

7.3 Interval Estimates of Lot Quality

7.3.1 *Confidence Interval for Lot Proportion Defective* (p') — In a sample of size n, if d defectives are found, then the confidence limits for the proportion of defectives in the lot are given by the two entries at the intersection of the row corresponding to n and the column corresponding to d in Table 3 (*see* P 50). For example, if in a sample of 20 spark plugs selected from a lot and inspected for high-tension current leakage, 2 were found not passing the test, then the estimated proportion of defective spark plugs in the lot is given by $\frac{2}{20} = 0{\cdot}10$ (i.e. 10 percent). Also the 95 percent confidence limits as read from Table 3 are 0·018 and 0·293 (i.e. 1·8 percent to 29·3 percent).

The confidence limits given in Table 3 are correct to three places of decimal and are for $n = 1\ (\ 1\ )\ 30$ and $d = 0\ (\ 1\ )\ \left[\frac{n}{2}\right]$. If d is greater than $\left(\frac{n}{2}\right)$, $(n-d)$ would be less than $\left(\frac{n}{2}\right)$, and the table can be read for confidence limits for the complementary proportion $(1-p)$ from which the confidence limits for p are obtained.

Example:

Suppose, $n = 25$ and $d = 14$. Then $(n-d) = 11$ and the 95 percent confidence limits for $(1-p)$ are given as $(0{\cdot}238, 0{\cdot}664)$, which means that the 95 percent limits for p would be $(1-0{\cdot}664,\ 1-0{\cdot}238) = (0{\cdot}336, 0{\cdot}762)$.

For sample sizes larger than 30, the confidence limits for the lot proportion of defectives may be calculated as: $\frac{1}{n}\left\{d + 2\sqrt{\frac{d(n-d)}{n}}\right\}$ and $\frac{1}{n}\left\{d - 2\sqrt{\frac{d(n-d)}{n}}\right\}$ respectively.

7.3.2 *Confidence Interval for the Average Count of Defects per Item in the Lot* ($\bar{c}$) — In a sample size n, if a total of c defects are counted, then the confidence limits for the average count of defects per item in the lot is obtained by dividing the entries against c in Table 4 (*see* P 52) by the corresponding sample size n.

Example:

In order to test the anti-corrosive character of a certain paint, 30 steel panels of the same area were coated with the paint drawn from different parts of the bulk supply and subjected to conditions likely to produce corrosion. The number of rust stains on each panel was counted. The total number of stains on 30 panels added up to 14.

Hence, the average stain per sample is obtained as $\bar{c} = \frac{14}{30} = 0{\cdot}47$. Also the 95 percent confidence limits for the average number of stains per panel in the lot is obtained as $\frac{7{\cdot}65}{30} = 0{\cdot}25$ and $\frac{23{\cdot}49}{30} = 0{\cdot}78$. Hence, the lower and upper confidence limits for average number of stains per panel ($= 0{\cdot}47$) are 0·25 and 0·78 respectively.

In case the number of defects counted (c) in a sample of size n becomes more than 50, then the confidence limits for the average count of defects per item in the lot may be calculated as:

$\left\{\bar{c} + 2\sqrt{\frac{\bar{c}}{n}}\right\}$ and $\left\{\bar{c} - 2\sqrt{\frac{\bar{c}}{n}}\right\}$ respectively.

7.3.3 *Confidence Interval for Lot Average Using s or R or $\bar{R}$* — If the measurements on n items of a sample are obtained as x_1, ... x_n leading to a sample average of $\bar{x}$, then the upper and lower confidence limits for the lot average are given by ($\bar{x} + h's$) and ($\bar{x} - h's$) where s is the sample standard deviation and h' is a factor corresponding to the sample size n obtained from Table 5 (*see* P 53). Alternatively, the confidence limits for the lot average may also be computed using the sample range (R) or sample mean range ($\bar{R}$) in place of sample standard deviation s in which case the necessary factor h shall be taken from Table 6 (*see* P 54) and the limits shall be ($\bar{x} + hR$) and ($\bar{x} - hR$) or ($\bar{x} + h\bar{R}$) and ($\bar{x} - h\bar{R}$).

The use of R is recommended when the sample size is less than 20, whereas $\bar{R}$ is to be used for sample sizes of 20 or more. While using $\bar{R}$ the grouping should be done in such a manner that:

a) each group has the same size,

b) the grouping of observations shall be in the natural order of obtaining the observations, and

c) the maximum group size shall not exceed 15.

Example:

With a view to estimating the mean breaking strength of 0·4 percent carbon steel produced in a plant, 12 specimens were tested and the values of their breaking strength (expressed in kg/cm^2) were as follows:

5 780
5 890
5 690
5 950
5 770
5 830
5 750
5 910
5 980
5 760
5 970
5 730

The arithmetic mean (= 5 834 kg/cm^2) would give a single value estimate for μ, the average breaking strength of the steel. If a confidence interval is desired in this case, it is necessary to have in addition a measure of variability.

a) Using 's'

If it is preferred to calculate the standard deviation,

$$s = \sqrt{\frac{408\,563\,300 - \dfrac{(\,70\,010\,)^2}{12}}{11}} = 101 \text{ kg/cm}^2$$

Referring to Table 5, the value of h' for $n = 12$, for determining confidence limits for the mean breaking strength in the lot is 0·635, so that the limits are:

$$(5\,834 - 0{\cdot}635 \times 101) = 5\,770 \text{ kg/cm}^2\text{, and}$$
$$(5\,834 + 0{\cdot}635 \times 101) = 5\,898 \text{ kg/cm}^2.$$

b) Using 'R'

If, on the other hand, it is desired to use range instead of standard deviation,

$$R = 5\,980 - 5\,690 = 290 \text{ kg/cm}^2$$

and Table 6 gives for $n = 12$ and $m = 1$, the value of $h = 0{\cdot}19$, so that the confidence limits would be:

$$(5\,834 - 0{\cdot}19 \times 290) = 5\,779 \text{ kg/cm}^2\text{, and}$$
$$(5\,834 + 0{\cdot}19 \times 290) = 5\,889 \text{ kg/cm}^2.$$

c) Using '$\bar{R}$'

Although the use of mean range based on sub-sample is not essential unless the gross sample contains more than 20 items, the same example may be used to illustrate the use of mean range. If the 12 items in the sample were considered to be grouped into two sub-samples of 6 items each, the first 6 measurements constituting one sub-sample and the remaining 6 the other sub-sample, then:

R for 1st sub-sample is (5 950 − 5 690) = 260 kg/cm², and
R for 2nd sub-sample is (5 980 − 5 730) = 250 kg/cm²,

$$\text{so that } \bar{R} = \frac{260 + 250}{2} = 255 \text{ kg/cm}^2.$$

Referring to Table 6 with $n = 6$ and $m = 2$ the value of $h = 0{\cdot}15$ is obtained. The confidence limits are then given by:

$$(5\,834 - 0{\cdot}25 \times 255) = 5\,770 \text{ kg/cm}^2\text{, and}$$
$$(5\,834 + 0{\cdot}25 \times 255) = 5\,898 \text{ kg/cm}^2.$$

NOTE — Limits obtained for the same lot quality may differ to some extent when different sample qualities are used. It should be noted that the meaning given to confidence intervals in **7.0.1** apply to a uniform procedure (using the same sample quality) and on an average basis. In the long run, but not necessarily in a particular case standard deviation would give shorter intervals than the range.

7.4 Interval Estimate of Lot Dispersion

7.4.1 *Confidence Interval for the Lot Standard Deviation, σ Using s or R* — Either the sample standard deviation (s) or sample range (R) may be used for obtaining the confidence limits for the lot standard deviation. When the former is used, the limits are obtained as $f_1 s$ and $f_2 s$ where f_1 and f_2 are the factors corresponding to the sample size n taken from Table 7 (*see* P 55). When the range R is used, the limits are obtained as $f_3 R$ and $f_4 R$ where f_3 and f_4 are the factors corresponding to sample size n taken from Table 8 (*see* P 56). The use of R is recommended only for small values of the sample size, say less than 20 or so.

Example:

In the example given under **7.3.3** for $n = 12$, Table 7 provides the values 0·708 and 1·70 for f_1 and f_2 to be used with s, and Table 8 provides the values 0·203 and 0·531 for f_3 and f_4 to be used with R. Thus, confidence limits for σ are given by:

$$\left.\begin{aligned} 0{\cdot}708 \times 101 &= 71{\cdot}5 \text{ kg/cm}^2 \\ \text{and } 1{\cdot}70 \times 101 &= 171{\cdot}7 \text{ kg/cm}^2 \end{aligned}\right\} \text{when } s \text{ is used}$$

$$\left.\begin{aligned} 0{\cdot}203 \times 290 &= 58{\cdot}9 \text{ kg/cm}^2 \\ \text{and } 0{\cdot}531 \times 290 &= 154{\cdot}2 \text{ kg/cm}^2 \end{aligned}\right\} \text{when } R \text{ is used.}$$

7.5 The Effect of Lot Size on Confidence Interval

7.5.1 In all the earlier paragraphs where the construction of confidence limits for different lot parameters have been discussed, the actual size of the lot had not been taken into account. This has been done since the effect of lot size on the length of the confidence interval is negligible only when sample size is small as compared to lot size (*see* Fig. 2). Since, in most of the practical situations, the sizes of the lots encountered are considerable as compared to the sample sizes, the tables given are readily applicable in all such cases.

7.6 Other Uses of Confidence Intervals

7.6.1 Besides providing an interval which contains the lot parameter, in the long run, the confidence limits are also useful for many other purposes. For example, while planning experiments, a rough idea of the lot quality together with the tables for construction of confidence limits can be used to determine how large the sample should be in order to obtain estimates of desired reliability. For further details of the actual methods, *see* IS : 5002-1969*. The confidence limits are also useful for testing certain hypothesis about population parameters.

---

*Methods for determination of sample size to estimate the average quality of a lot or process.

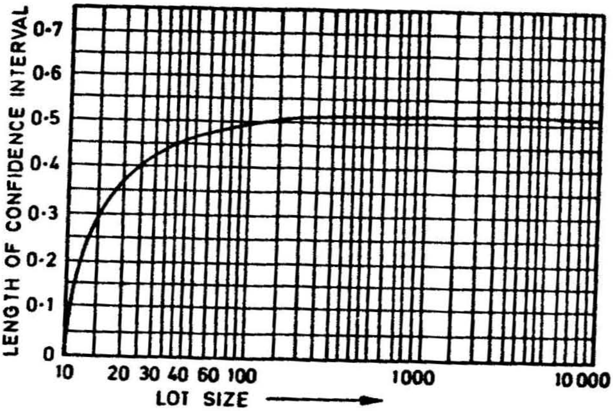


FIG. 2 LENGTH OF CONFIDENCE INTERVAL FOR p CONSTRUCTED FROM $n = 10$, $d = 2$ FOR DIFFERENT LOT SIZES

8. SAMPLING FOR LOT ACCEPTANCE

8.1 Risks Involved in Sampling Inspection — In sampling for lot acceptance, a decision is arrived at about the disposal of the lot as a whole as acceptable or not acceptable on the basis of inspection of sample items. In such a process, wrong decisions by way of either rejecting good lots against the interests of the supplier (or producer) or accepting bad lots against the interests of the receiver (or consumer) are possible. The consumer and producer will, therefore, have to take some risks. It also follows that in lots accepted on a sampling basis, some defective items would be present.

8.1.1 *Operating Characteristic* (*OC*) — When an acceptance plan operates and a number of lots are inspected, the result is to sort out the lots into two groups, namely, the accepted and non-accepted groups. The relative proportions into which lots will be sorted out by any particular plan as acceptable or not will, along with other factors, depend upon the quality of the lots coming for inspection. The quality of incoming lots might vary. The different proportions into which a plan will, on an average, be able to sort the lots at varying incoming quality levels would, therefore, give a picture of the degree of quality protection given by the plan. For instance, if the consumer fixes a certain level of quality for lots to be considered bad, he can know from the proportion of lots that would be accepted at that quality, whether a given plan satisfies a stipulated risk of accepting bad lots or not. The performance of a plan, measured in terms of proportion of the lots that would be accepted, on an average, at different possible incoming qualities, is called the operating characteristic (OC) of the plan. Each acceptance sampling plan will thus have its own OC. The OC of a plan can be expressed as a mathematical function of the incoming lot quality. The graph of this function with incoming lot quality on the horizontal axis is called the OC-curve. If the incoming material is free of any defectives, then the lot will always be accepted, whatever may be the sampling plan chosen. On the other hand, if it contains only defective material, the lot will always be rejected, irrespective of the type of sampling plan chosen. Hence, the OC-curve would continuously slope downwards from the value one to zero in the direction of deterioration of quality, indicating that the plan will have lesser and lesser preference for worse and worse incoming lot qualities. The ability of a plan to discriminate between good and bad lots can be judged from the steepness with which the OC-curve slopes downwards. OC of a plan, effectively summarizes the nature of quality protection obtainable by operating the plan on a number of lots. OC, thus, plays an important role in the choice of a plan and many of the published plans also furnish OC-curves to enable choice from the point of view of quality protection.

8.1.1.1 A typical OC-curve is shown in Fig. 3, in which the incoming lot quality is expressed as percentage defective (the larger the percentage defective, the worse is the quality). The OC-curve may be regarded as a graph of the percentage of lots expected to be accepted in the long run in sampling inspection plotted against incoming lot quality. If a certain value is chosen to demarcate good lots and another to demarcate bad lots, it is easy to identify the producer's and consumer's risk on the OC-curve as shown in Fig. 3. The value demarcating good lots is called the acceptable quality level (AQL) and the value demarcating bad lots is lot tolerance percent defective (LTPD). LTPD is also designated as limiting quality level (LQL). The producer wants all good lots accepted and the consumer wants all bad lots rejected. Only an ideal OC-curve, as shown in Fig. 4. can achieve such an objective. However, sampling plans can have OC-curves close to the ideal OC only, thereby implying that certain risks for both the producer and the consumer are inevitable. In Fig. 3, lots with percentage defective equal to AQL or less will be considered good. Lots with quality equal to LTPD or worse will be considered bad. Even when the producer submits lots of AQL quality, he runs a small risk of such lots being rejected by the plan. This is usually known as producer's risk. Similarly, when the lots of LTPD quality are submitted, the consumer runs small risk of accepting such loss which is known as consumer's risk. With the shape of the OC-curve, as given in Fig. 3, it would be seen that the risk of rejecting lots of quality better than AQL will be less than the producer's risk. Similarly, the risk of accepting lots of quality worse than LTPD will be less than the consumer's risk. This is a very desirable feature common to most of the acceptance plans. In practice, producer's risk is normally taken as 5 percent whereas consumer's risk is kept as 10 percent.

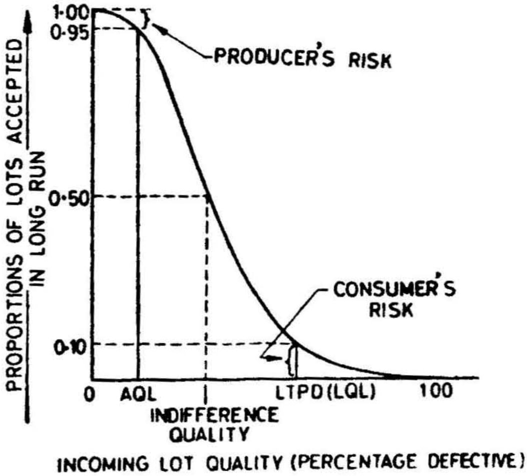


FIG. 3 AN OC-CURVE

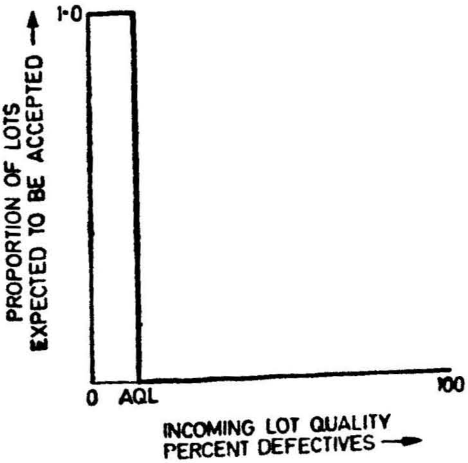


FIG. 4 AN IDEAL OC-CURVE

The lot quality, which has a 50 percent chance of being accepted or rejected by the plan, is also sometimes used as a measure of protection afforded by the plan. This lot quality is normally known as indifference quality.

NOTE — Though in the development of the theory of sampling inspection, AQL was associated with a fixed producer's risk (5 percent), the later trend has been to link it with the attainable process average. Thus, the lots, for which the process average level is not greater than the specified AQL, are accepted most of the times. Accordingly, the producer's risk is not set at a particular value. For further details, *see* IS : 2500 (Part I)-1973* and IS : 2500 (Part II)-1965†.

8.1.2 *Percentage Sampling* — A common practice in industry is to specify that the sampling inspected shall be some fixed percentage of the lot, such as, 5, 10 or 20 percent. This stipulation is generally based on the mistaken idea that the protection given by sampling plans is constant if the ratio of the sample size to lot size is kept constant. It can, however, be shown with the help of OC-curves that acceptance sampling plans with the same percentage samples give very different quality protections. For example, Fig. 5 gives OC-curves for 4 plans wherein the sample is 10 percent of the lot size (lots of sizes 50, 100, 200 and 1 000 are considered).

It can be seen from Fig. 5 that incoming lots of 2 percent defectives would have a very high chance (about 90 percent) of being accepted when the lots are of size 50 and a very poor chance (about 12 percent) of being accepted when the lots are of size 1 000.

8.1.3 *Fixed Sample Size* — As against the percentage sampling discussed in 8.1.2, it may be of interest to study the behaviour of the plans with fixed sample sizes. Figure 6 gives the OC-curves of 4 plans wherein a sample of size 20 is inspected (lots of sizes 50, 100, 200 and 1 000 are considered). It can be easily seen that the fixed sample size tends towards the constant quality protection, since the corresponding OC-curves are close to each other.

8.1.4 *Varying Sample Size* — Figure 7 illustrates the change in the shape of OC-curves for different sample sizes keeping the acceptance number constant. As the sample size increases, the slope of the curve becomes steeper and approaches straight vertical line. Sampling plans with large sample sizes are better able to discriminate between good and bad quality lots. Therefore, the consumer has fewer lots of bad quality accepted and the producer fewer lots of good quality rejected.

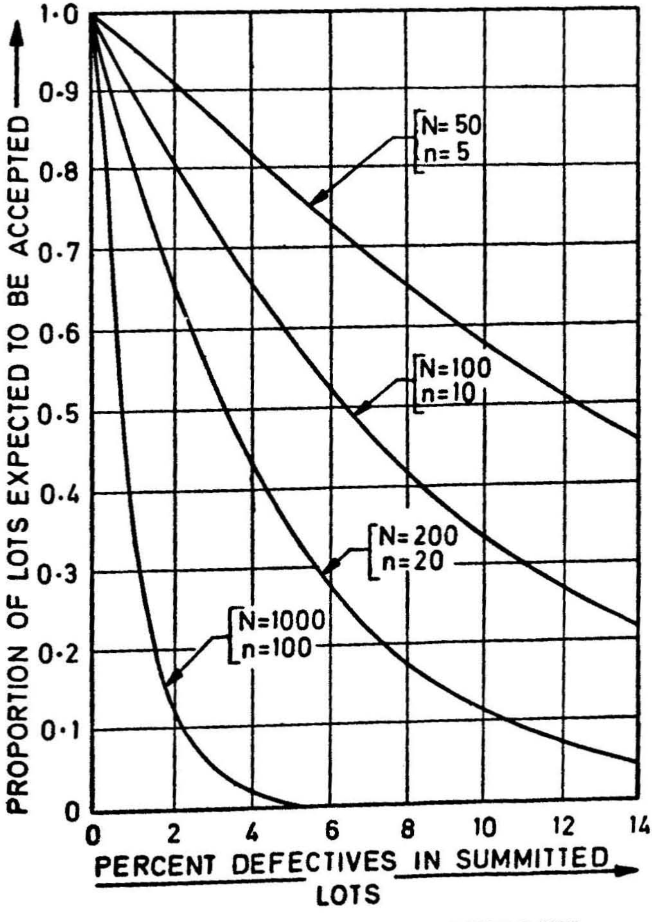
8.1.5 *Varying Acceptance Numbers* — The changes in the shape of OC-curves as the acceptance number changes keeping the sample size fixed are shown in Fig. 8. As the acceptance number decreases, the curve becomes

---

*Sampling inspection tables: Part I Inspection by attributes and by count of defects (*first revision*).

†Sampling inspection tables: Part II Inspection by variables for percent defective.

steeper. Consequently the OC-curve for plans with acceptance number zero are the steepest. This fact has frequently been incorrectly used to justify the use of sampling plans with zero acceptance number. It could, however, be seen that OC-curves of such plans are convex throughout the range of incoming quality, resulting in higher risks of rejecting good lots. As against



N = LOT SIZE n = SAMPLE SIZE

FIG. 5 SOME OC-CURVES FOR PERCENTAGE SAMPLING

this, OC-curve for $n = 200$ and $a = 1$, which is shown by dotted line in Fig. 8, is not only steeper than the OC-curve but also for the plan $n = 50$ and $a = 0$, is more concave at the lower values of incoming quality. Thus, the plan for $n = 200$ and $a = 1$ is more discriminating in the acceptance of AQL quality of incoming material and rejection of LTPD quality as compared to other plans. Sampling plans with acceptance numbers greater than zero can actually be superior to those with zero. Therefore, sampling plans with higher sample sizes and non-zero acceptance numbers may usually be considered better.

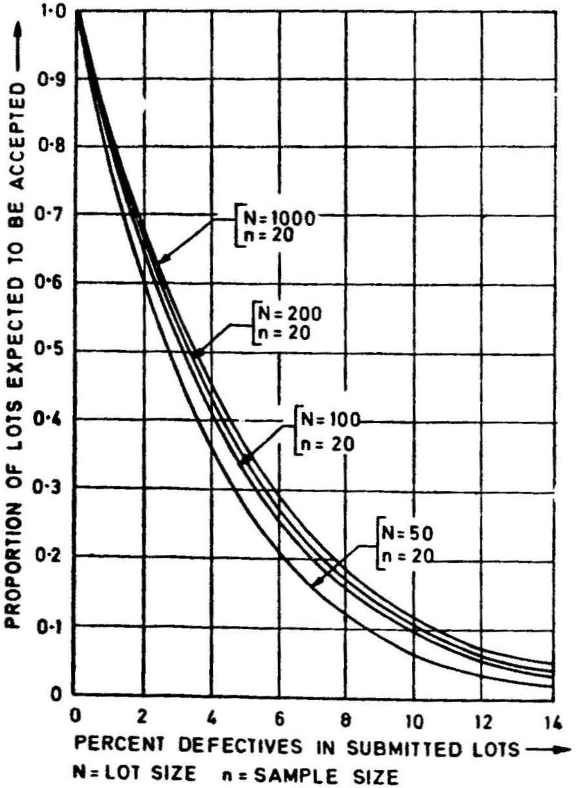


FIG. 6 SOME OC-CURVES FOR PLANS WITH CONSTANT SAMPLE SIZES

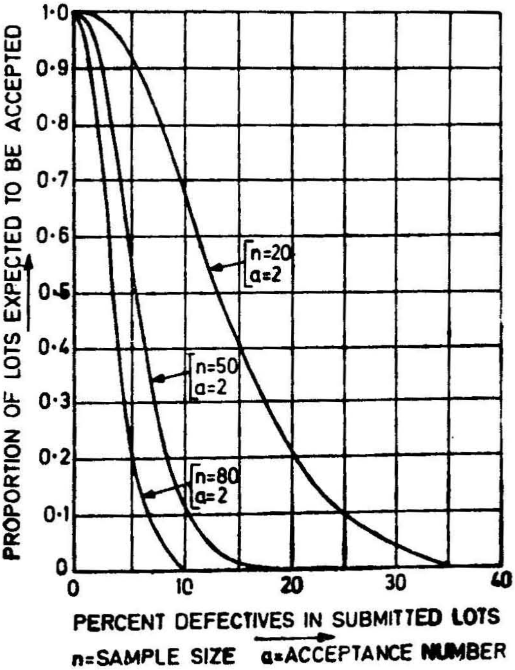


FIG. 7 OC-CURVES FOR VARYING SAMPLE SIZES

8.1.6 *Acceptance Numbers as a Fixed Percentage of Sample Size* — Figure 9 illustrates the change in the shape of OC-curves for three different sampling plans having the same ratio of acceptance number to sample size. The three plans are $\begin{bmatrix} n = 20 \\ a = 2 \end{bmatrix}$, $\begin{bmatrix} n = 50 \\ a = 5 \end{bmatrix}$, and $\begin{bmatrix} n = 200 \\ a = 20 \end{bmatrix}$.

Although each of these three plans permits 10 percent of the sample to be defective, it is evident that they have quite different OC-curves. The larger the sample size, the greater the ability of the sampling plan to discriminate between lots of different qualities. For example, a 15 percent defective lot, if submitted for inspection, will have a probability of acceptance as 40·5 percent for the plan $\begin{bmatrix} n = 20 \\ a = 2 \end{bmatrix}$, but is practically certain to be rejected for the plan $\begin{bmatrix} n = 200 \\ a = 20 \end{bmatrix}$. The large sample, which protects the consumer against

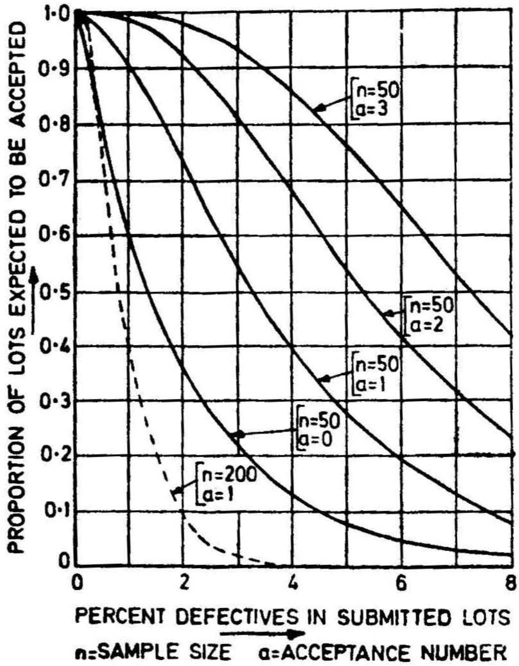


FIG. 8 OC-CURVES FOR VARYING ACCEPTANCE NUMBERS

the acceptance of relatively bad lots, also gives the producer better protection against the rejection of relatively good ones. For example, a 5 percent defective lot has probability of acceptance as 92·5 percent for $\begin{bmatrix} n = 20 \\ a = 2 \end{bmatrix}$ and 99·8 percent for $\begin{bmatrix} n = 200 \\ a = 20 \end{bmatrix}$.

8.2 Average Outgoing Quality Limit (AOQL) — The operating characteristic, while it provides a complete picture of the performance of plan in the form of a function or a curve, does not give a single value measure of the performance. In view of this, other concepts are sometimes used. One such is the average outgoing quality limit (AOQL).

8.2.1 When a plan operates on a number of lots, out of which some will be accepted the non-accepted lots may be rejected and returned or may be completely inspected for the non-sampled portion of each lot. In the latter case, all the defective items found would be replaced by the good ones. Accordingly, an acceptance plan may be either of the acceptance-rejection type or of the acceptance-rectification type. For both these types, there would be a certain resulting quality for all the accepted lots put together called the average outgoing quality (AOQ). This would naturally depend on the incoming lot quality. If the plan is of the acceptance-rejection type, then the AOQ will be almost the same as the incoming quality (the slight reduction in the value of AOQ as compared to the incoming lot quality is due to the fact that the defectives found in the samples of the accepted lots are eliminated). On the other hand, for an acceptance-rectification type of plan, the AOQ will be superior to the incoming quality since the non-accepted lots will be completely inspected, made free of defectives and then

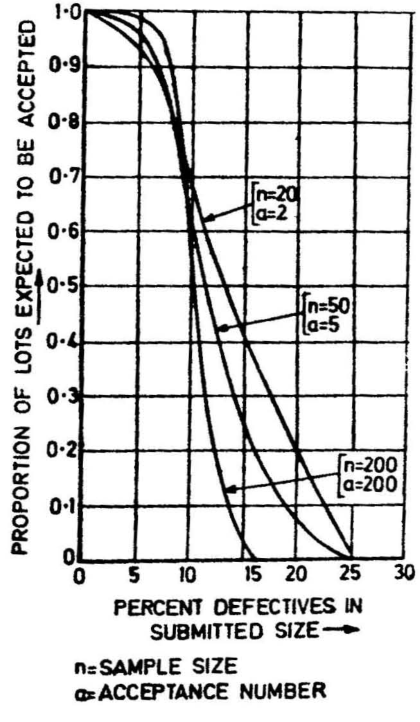


FIG. 9 OC-CURVES FOR CONSTANT VALUES OF $\left(\frac{a}{n}\right)$

added to the lots accepted at first. Hence, the AOQ concept is more meaningful in relation to the acceptance-rectification type of plans.

8.2.2 If the incoming lots are absolutely free of defectives, the value of AOQ would obviously be zero for any sampling inspection plan. Again, if the incoming lots contain only defectives then all the lots would be rejected by the sampling plan chosen and since rejected lots are to be screened for replacing the defectives by the good items, the value of AOQ would again be zero. For incoming lots of intermediary qualities, the value of AOQ would vary depending on the quality level of the incoming lots. A typical AOQ-curve for the plan in which 50 sample items are inspected from each lot and the lot is accepted if one or less defective is encountered is given in Fig. 10.

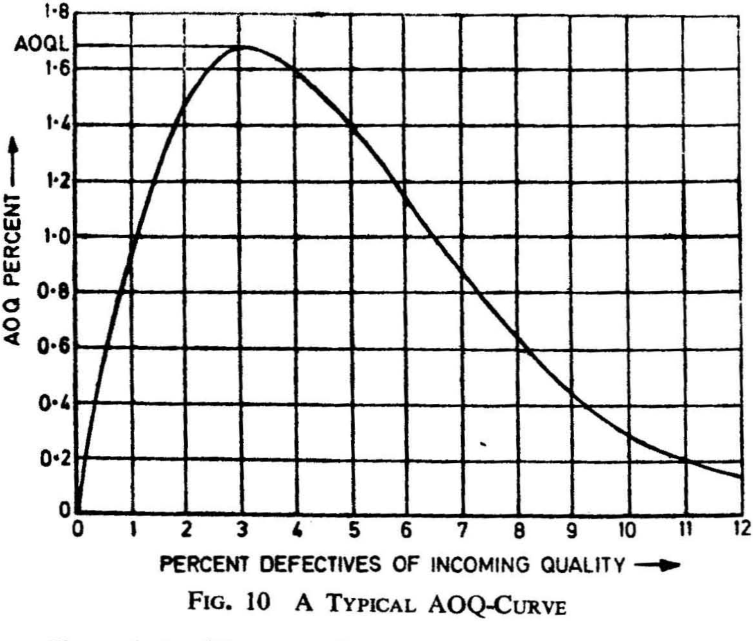


FIG. 10 A TYPICAL AOQ-CURVE

8.2.3 The analysis of the curve shows that when the incoming quality is 2 percent defective, the AOQ is 1·48 percent defective, and when the incoming quality is 8 percent defective, AOQ is 0·64 percent defective. Since the rejected lots are rectified, the AOQ is always better than the incoming quality. From Fig. 10 it can be easily seen that the AOQ cannot exceed a certain limit. This upper limit is called the average outgoing quality limit (AOQL). The AOQL of a plan is, therefore, a summary measure of the quality protection in the case of an acceptance-rectification plan. It should, however, be noted that the AOQL is only an upper limit of an average and it may, sometimes, be exceeded in individual cases. However, in the long run, the percent defectives in the accepted lots would never exceed the AOQL. Thus, an AOQ-curve, in conjunction with an OC-curve, provides a powerful tool for describing and analysing acceptance-rectification type of sampling plans.

8.3 Different Types of Sampling Plans

8.3.1 Depending upon whether the inspection is by gauging (or visual) or by measurement, the attributes or the variables type of plans can be used. For both these types, the lot quality may be expressed as a percentage defective (*see* **3.2.3**). Hence, most of the plans (both attributes and the variables type) are designed to suit lot quality specifications in terms of percentage defectives. In the attributes plan, the sample quality that is generally used is the number of defectives (or defects per item) found in the sample, whereas in the variables plan the sample quality that is generally used is the sample mean ($\bar{x}$) along with lot standard deviation (σ), if it has been determined and is known to be constant (stable) from lot to lot. In all situations, the sample quality used is the sample mean along with sample standard deviation (s), or the sample range (R) or the sample mean range ($\bar{R}$).

8.3.2 *Attributes Plans*

8.3.2.1 *Single sampling plan* — In this type of sampling plans, decision on the acceptance or rejection of the lot is always made on the evidence of only one sample taken from the lot. So a single sampling plan is defined by the lot size N, the sample size n, and the acceptance number a. If the number of defectives found in the sample is less than or equal to a, the lot is accepted; otherwise the lot is rejected. Hence, rejection number in this case is given by ($a + 1$) (also denoted by r).

8.3.2.2 *Double sampling plan* — In this type of sampling plans, one sample is taken from each lot and the evidence is used to accept the lot, or to reject it or to reserve decision until further information from a second sample is obtained. Thus, one sample from each lot is always taken and never more than two samples are taken from the lot. So, a double sampling plan is defined by:

N = Lot size

n = Size of the first or second sample

a_1 = Acceptance number for the first sample

r_1 = Rejection number for the first sample

a_2 = Acceptance number for the first and second samples combined

r_2 = Rejection number for the first and second samples combined ($= a_2 + 1$)

Schematically, a double sampling acceptance-rejection plan can be presented as shown in Fig. 11.

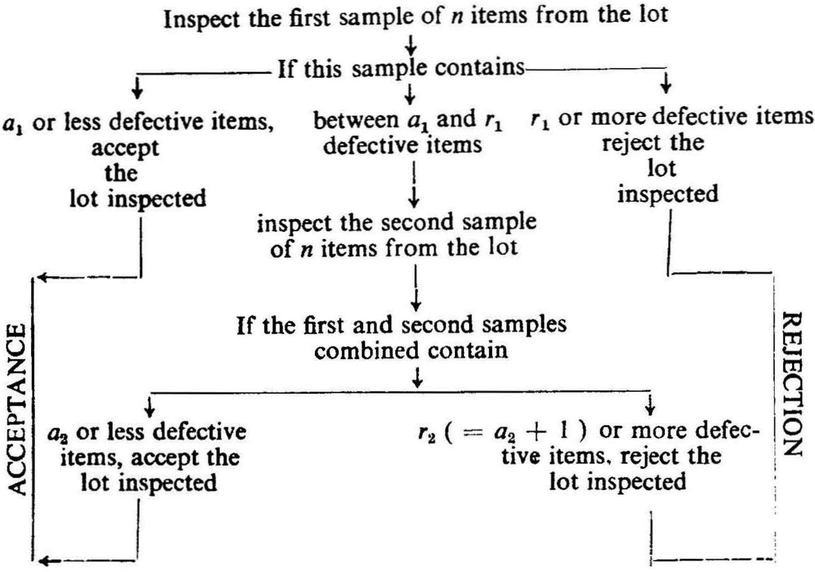

Inspect the first sample of n items from the lot
If this sample contains
a_1 or less defective items, accept the lot inspected
between a_1 and r_1 defective items
r_1 or more defective items, reject the lot inspected
inspect the second sample of n items from the lot
If the first and second samples combined contain
a_2 or less defective items, accept the lot inspected
r_2 ($= a_2 + 1$) or more defective items, reject the lot inspected
ACCEPTANCE
REJECTION

FIG. 11 SCHEMATIC REPRESENTATION OF A DOUBLE SAMPLING PLAN

8.3.2.3 *Multiple sampling plan* — A multiple sampling plan is a continuation of the double sampling plan. In this type of plan, a certain number of samples, not exceeding a maximum number of stages specified, are taken before taking a decision on acceptance or rejection of the lot. Although it may be possible to devise sampling plans for any number of stages, the plans published in IS : 2500 (Part I)-1973* are given up to seven stages.

8.3.2.4 *Sequential sampling plans* — While in multiple sampling a decision is attempted after each sample is inspected and a decision is arrived at in any case with the last allowable sample, in the sequential type

---

*Sampling inspection tables: Part I Inspection by attributes and by count of defects (*first revision*).

of inspection a decision is attempted on the cumulative evidence gathered at each stage of item-by-item or group-by-group inspection of items. This type of inspection is continued till a categoric decision to accept or reject the lot is possible. There is, therefore, no question of pre-determined sample sizes for sequential plans. Sequential sampling plans lead, on an average, to the least amount of inspection among all known attributes plans.

Simple formulae based on some stipulations are available for item-by-item sequential plans, in which the decision to accept, reject or to continue sampling is made after inspection of each item. The plan can also be operated graphically with the number of items inspected at any stage represented on one axis and the total number of defectives found in the sample on the other axis. The plan is determined by an acceptance line $a_n = - m_1 + nS$ and a rejection line $r_n = m_2 + nS$, where m_1, m_2 and S would depend on the lot qualities and acceptance risks stipulated beforehand.

8.3.2.5 For the relative merits of the various attributes plans discussed above as also for obtaining the actual value of sample size, acceptance number, etc, relevant to the plans for different stipulations, *see* IS : 2500 (Part I)-1973*.

8.3.2.6 *Examples* — Suppose lots containing 2 000 mild steel tubes each are submitted for inspection of outside diameters. The outside diameters of the tubes should be between 9·8 mm and 10·6 mm. Any tube failing to meet this requirement is considered as defective and suitable gauges are used for the detection of defectives.

a) If a single sampling plan for an AQL of 4 percent is adopted, the plan may be as follows:

Sample size (n)=125
Acceptance No. (a)=10

From each lot of 2 000 tubes, take at random a sample of 125 tubes and inspect them for outside diameters using Go, No-Go gauges. The lot shall be accepted if the number of defective tubes found in the sample is 10 or less; otherwise it shall be rejected.

b) If a double sampling plan is adopted, the plan would be as follows:

Sample Stage	*Sample Size* (n)	*For Cumulative Sample*		
		Size	Acceptance No. (a)	Rejection No. (r)
First	80	80	5	9
Second	80	160	12	13

---

*Sampling inspection tables: Part I Inspection by attributes and by count of defects (*first revision*).

From each lot of 2 000 tubes, collect the first sample of 80 tubes at random and examine them for defectives. The lot shall be accepted if the number of defective tubes in the first sample is 5 or less and rejected if it is 9 or more. If the number of defective tubes is between 5 and 9 (i.e. 6, 7 or 8) a second sample of 80 tubes shall be collected and inspected. The lot shall be accepted if the number of defective tubes in the combined sample of 160 tubes is 12 or less and rejected if the number of defective tubes found is 13 or more.

c) If instead of a double sampling plan, a multiple sampling plan is desired to be followed, the sampling plan would be as follows:

Sample Stage	*Sample Size* (n)	*For Cumulative Sample*		
		Size	Acceptance Number (a_i)	Rejection Number (r_i)
1	32	32	0	5
2	32	64	3	8
3	32	96	6	10
4	32	128	8	13
5	32	160	11	15
6	32	192	14	17
7	32	224	18	19

From each lot collect the first sample of 32 tubes in a random order and examine them for defective tubes. The lot shall be accepted if no defective tube is found. It shall be rejected if the number of defective tubes found is 5 or more. If, however, the number of defective tubes is between 0 and 5, a second sample of 32 tubes shall be selected and examined for defectives. The number of defective tubes in the combined sample (that is, the first and the second sample taken together) shall then be compared against the acceptance and rejection number corresponding to the second stage of sampling for taking decision with regard to the acceptance or rejection of the lot. If no decision is reached after examining the second sample, a third sample of 32 tubes shall be drawn and so on up to the seventh stage till a decision is finally reached so as to accept or reject the lot.

8.3.3 *Variables Plans*

8.3.3.1 *Single sampling plans* — The procedure in single sampling variables plan would be as follows:

Obtain the measurements on n items in the sample. Calculate the arithmetic mean ($\bar{x}$) of n measurements. Also, if σ is not known, calculate the sample standard deviation (s) or sample range (R) or mean range ($\bar{R}$). (When the sample size is 10 or more, $\bar{R}$ shall be calculated from the ranges obtained for sub-groups of size 5 each).

If ($\bar{x} + k''\sigma$) or ($\bar{x} + k's$) or ($\bar{x} + kR$) or ($\bar{x} + k\bar{R}$), whichever is the case, is less than or equal to U, accept the lot otherwise reject, when only the upper specification limit U is stipulated. For a lower limit L, if ($\bar{x} - k''\sigma$) or ($\bar{x} - k's$) or ($\bar{x} - kR$) or ($\bar{x} - k\bar{R}$) is greater than or equal to L, the lot is accepted, otherwise rejected. The values of the factors k'', k' and k to suit different stipulations are available in published tables.

For obtaining the actual values of the sample size and the relevant factor k (or k' or k'') for plans with various stipulations, *see* IS : 2500 (Part II)-1965*.

When the item quality specification is two-sided (that is, in terms of both L and U), one of the methods available is to use two one-sided plans separately, one for each limit using the same k values as of the one-sided plans provided the value of σ or s or R or $\bar{R}$ is quite small when compared to the specification range ($U - L$). When this is not the case, the acceptance procedure is graphical and quite cumbersome. However, this cumbersome graphical procedure has been considerably simplified by imposing an upper limit on the value of $\frac{\sigma}{U-L}$ or $\frac{s}{U-L}$ or $\frac{R}{U-L}$ or $\frac{\bar{R}}{U-L}$. [For further details, *see* IS : 2500 (Part II)-1965*.]

8.3.3.2 *Double sampling plans* — The principle underlying the double sampling variables plan is similar as the one indicated for the corresponding attributes plan. However, in practice, these types of plans are very cumbersome to operate and, hence, do not have many uses in industrial applications.

8.3.3.3 *Sequential plans* — Sequential plans are also possible in case of variables inspection which requires the determination of acceptance and rejection lines as in the case of attributes sequential plans.

8.3.3.4 If for the example given in **8.3.2.6**, a single sampling variables plan for an AQL of 4 percent is to be adopted, the sampling plan for variability known and variability unknown would be as follows:

a) *Variability known method*

Sample size $= 15$
Factor k'' $= 1{\cdot}35$

---

*Sampling inspection tables: Part II Inspection by variables for percent defective.

Upper limit of $\frac{\sigma}{U - L} = 0{\cdot}243$, where U and L are upper and lower specification limits.

Take a sample of 15 items at random from each lot and find the outside diameter of each of the sample tubes. Compute the mean ($\bar{x}$) of these 15 observations. The lot shall be accepted if all the following conditions are satisfied; otherwise it shall be rejected:

i) $\frac{\sigma}{0{\cdot}8} \leqslant 0{\cdot}243$

ii) $\bar{x} + 1{\cdot}35\ \sigma \leqslant 10{\cdot}6$ (upper specification limit)

iii) $\bar{x} - 1{\cdot}35\ \sigma \geqslant 9{\cdot}8$ (lower specification limit)

NOTE — The value of σ is required to be known beforehand from past experience.

b) *Variability unknown (standard deviation) method*

Sample size = 30

Factor k' = 1·36

Upper limit of $\frac{s}{U - L} = 0{\cdot}295$

Take a sample of 30 items at random from each lot and find the outside diameter of each of the sample tubes. Compute the mean ($\bar{x}$) and standard deviation (s) of these 30 observations. The lot shall be accepted if all the following conditions are satisfied; otherwise the lot shall be rejected:

i) $\frac{s}{0{\cdot}8} \leqslant 0{\cdot}295$

ii) $\bar{x} + 1{\cdot}36\ s \leqslant 10{\cdot}6$

iii) $\bar{x} - 1{\cdot}36\ s \geqslant 9{\cdot}8$

c) *Variability unknown (range) method*

Sample size = 35

Factor k = 0·581

Upper limit of $\frac{R}{U - L}$ or $\frac{\bar{R}}{U - L} = 0{\cdot}694$

Take a sample of 35 tubes at random from each lot and find outside diameter of each of the sample tubes. Compute the mean ($\bar{x}$) and mean range ($\bar{R}$) of these 35 observations. The lot shall be accepted if all the following conditions are satisfied; otherwise it shall be rejected:

i) $\frac{\bar{R}}{0.8} \leqslant 0.694$

ii) $\bar{x} + 0.581\,\bar{R} \leqslant 10.6$

iii) $\bar{x} - 0.581\,\bar{R} \geqslant 9.8$

8.4 Average Amount of Inspection

8.4.1 The cost of inspection involved in using a sampling plan would depend on several factors, such as the cost of transporting items to make them available for inspection, the type of inspection performed (visual, measurement, etc) and, above all, the number of items required to be inspected. For comparison of costs involved in using different plans, however, often the average amount of inspection per lot (AOI) or the average sample number (ASN) is used. ASN is the average number of items inspected per lot in the sampling inspection whereas AOI is the average number of items inspected per lot in sampling as well as in sorting inspection. ASN and AOI are same in acceptance-rejection plans but are much different in acceptance-rectification plans. In a single sampling plan, with sample size n, the ASN is equal to n if the plan is of the acceptance-rejection type. The corresponding ASN for a double sampling plan would be of the form $n (1 + f)$ where n is the size of the first or second sample and f the fraction of lots that will be subjected to a second inspection. (These average sample numbers hold good if there is no curtailment of inspection, i.e. the sample selected is completely inspected, irrespective of the decision to accept or reject the lot).

8.4.1.1 Typical ASN-curves for single, double and multiple sampling plans given in **8.3.2.6** having same OC-curves are given in Fig. 12.

8.4.1.2 In a single sampling plan (acceptance-rectification type) the AOI $= n + F (N - n)$ where n is the sample size, N the lot size and F the fraction of lots that will not be accepted in the first instance but will be accepted after complete inspection and rectification.

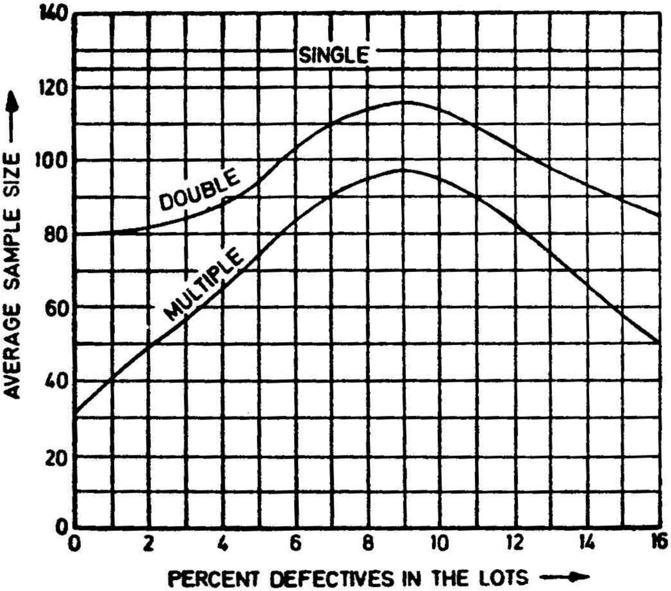


FIG. 12 TYPICAL ASN-CURVES

APPENDIX A

(*Clause* 2.0)

GLOSSARY OF SYMBOLS

a, (a_1, a_2.....)	— acceptance number in the single sampling plan or in the first sample of double sampling plans; a_1, a_2, etc, are used for denoting the acceptance numbers at various stages of multiple sampling plans.
AOI	— average amount of inspection per lot.
AOQ	— average outgoing quality.
AOQL	— average outgoing quality limit.

AQL — acceptable quality level.

ASN — average sample number.

c — the number of defects.

$\bar{c}$ — the average count of defects per item in a sample.

$\bar{c'}$ — the average count of defects per item in a lot.

c_1 — coefficient of s for estimating σ. Values of c_1 are given in Table 1.

d — number of defective items in a sample.

d_2 — coefficient of R or $\bar{R}$ for estimating σ. Values of d_2 are given in Table 2.

f_1 and f_2 — coefficients of s for estimating the confidence limits of σ. Values of f_1 and f_2 are given in Table 7.

f_3 and f_4 — coefficients of R for estimating the confidence limits of σ. Values of f_3 and f_4 are given in Table 8.

h — coefficient of R or $\bar{R}$ for estimating the confidence limits of μ. Values of h are given in Table 6.

h' — coefficient of s for estimating the confidence limits of μ. Values of h' are given in Table 5.

k, k', k'' — coefficients of R, s and σ respectively in the acceptance/rejection criteria for single sampling plan by variables.

L — lower specification limit.

LTPD — lot tolerance percent defective.

LQL — limiting quality level.

m_1, m_2 — the distance of acceptance-rejection lines from the origin on the y-axis in the case of sequential sampling plans.

n — sample size.

N — lot size.

OC — operating characteristic.

OC Curve — operating-characteristic curve.

p — sample fraction defective ($=d/n$).

p' — lot fraction defective.

r (r_1, r_2...) — the rejection number in single and double sampling plans. r_1, r_2, ... etc, are used for denoting the rejection number at various stages of multiple sampling plans.

R — the range.

$\bar{R}$ — the mean range.

s — sample standard deviation. If $x_1, x_2, \ldots.. x_n$ are the n measurements of items in the sample, then

$$s = \sqrt{\frac{(x_1 - \bar{x})^2 + \ldots + (x_n - \bar{x})^2}{n - 1}}$$

$$= \sqrt{\frac{(x_1^2 + \ldots.. x_n^2) - n\bar{x}^2}{n - 1}}$$

S — the slope of the acceptance-rejection lines in the case of sequential sampling plans.

σ — lot standard deviation. If $x_1, x_2, \ldots.. x_N$ are the N measurements of the items in a lot, then

$$\sigma = \sqrt{\frac{(x_1 - \mu)^2 + \ldots + (x_N - \mu)^2}{N}}$$

$$\sigma = \sqrt{\frac{(x_1^2 + \ldots + x_N^2) - N\mu^2}{N}}$$

U — upper specification limit.

$\bar{x}$ — sample average. If $x_1, x_2 \ldots\ldots x_n$ are the n measurements of the items in a sample, then

$$\bar{x} = \frac{x_1 + x_2 + \ldots.. + x_n}{n}$$

μ — lot average. If $x_1, x_2, \ldots.. x_N$ are the N measurements of items in a lot, then

$$\mu = \frac{x_1 + x_2 + \ldots.. + x_N}{N}$$

v — coefficient of variation. In the case of the sample this is given by $\frac{s}{\bar{x}} \times 100$ (expressed as a percentage), whereas in the case of the lot it is obtained as $\frac{\sigma}{\mu} \times 100$ (expressed as a percentage).

TABLE 1 FACTORS NECESSARY FOR ESTIMATING σ FROM s

(*Clause* 7.2.1)

Sample Size n	Factor c_1	Sample Size n	Factor c_1
2	0·797 8	12	0·977 0
3	0·886 4	13	0·979 6
4	0·921 6	14	0·981 2
5	0·939 9	15	0·982 2
6	0·951 1	16	0·983 7
7	0·959 3	17	0·984 7
8	0·965 0	18	0·985 4
9	0·969 6	19	0·985 8
10	0·972 5	20	0·986 9
11	0·975 6		

NOTE — σ is estimated by $\frac{s}{c_1}$.

TABLE 2 FACTORS NECESSARY FOR ESTIMATING σ FROM R OR $\overline{R}$

(*Clause* 7.2.1.1)

Sample Size n	Factor d_2	Sample Size n	Factor d_2
2	1·128	12	3·258
3	1·693	13	3·336
4	2·059	14	3·407
5	2·326	15	3·472
6	2·534	16	3·532
7	2·704	17	3·588
8	2·847	18	3·640
9	2·970	19	3·689
10	3·078	20	3·735
11	3·173		

NOTE — σ is estimated by $\frac{R}{d_2}$ or $\frac{\overline{R}}{d_2}$.

TABLE 3 CONFIDENCE LIMITS FOR PROPORTION OF DEFECTIVE IN A LOT
(CONFIDENCE COEFFICIENT 95 PERCENT)

(*Clause* 7.3.1)

n	*d* = 0	*d* = 1	*d* = 2	*d* = 3	*d* = 4	*d* = 5	*d* = 6	*d* = 7	*d* = 8	*d* = 9	*d* = 10	*d* = 11	*d* = 12	*d* = 13	*d* = 14	*d* = 15
1	·950 ·000															
2	·776 ·000	·975 ·025														
3	·632 ·000	·865 ·017														
4	·527 ·000	·751 ·013	·902 ·098													
5	·500 ·000	·657 ·010	·811 ·076													
6	·402 ·000	·598 ·009	·729 ·063	·847 ·153												
7	·377 ·000	·554 ·007	·659 ·053	·775 ·129												
8	·315 ·000	·500 ·006	·685 ·046	·711 ·111	·807 ·193											
9	·289 ·000	·443 ·006	·558 ·041	·711 ·098	·749 ·169											
10	·267 ·000	·397 ·005	·603 ·037	·619 ·087	·733 ·150	·778 ·222										
11	·250 ·000	·369 ·005	·500 ·033	·631 ·079	·667 ·135	·750 ·200										
12	·236 ·000	·346 ·004	·450 ·030	·550 ·072	·654 ·123	·706 ·181	·764 ·236									
13	·225 ·000	·327 ·004	·434 ·028	·520 ·066	·587 ·113	·673 166	·740 ·224									
14	·206 ·000	·312 ·004	·389 ·026	·500 ·061	·611 ·104	·629 ·153	·688 ·206	·794 ·206								
15	·191 ·000	·302 ·003	·369 ·024	·448 ·057	·552 ·097	·631 ·142	·668 ·191	·706 ·191								

n	*d* = 0	*d* = 1	*d* = 2	*d* = 3	*d* = 4	*d* = 5	*d* = 6	*d* = 7	*d* = 8	*d* = 9	*d* = 10	*d* = 11	*d* = 12	*d* = 13	*d* = 14	*d* = 15
16	·178	·272	·352	429	·500	·571	·648	·728	·728							
	·000	·003	·023	·053	·090	·132	·178	·178	·272							
17	·166	·254	·337	·417	·489	·544	·594	·663	·746							
	·000	·003	·021	·050	·085	·124	·166	·166	·253							
18	·157	·242	·325	·381	·444	·556	·619	·625	·675	·758						
	·000	·003	·020	·047	·080	·116	·156	·157	·236	·242						
19	·150	·232	·316	·365	·426	·500	·574	·635	·655	·688						
	·000	·003	·019	·044	·075	·110	·147	·150	·222	·232						
20	·143	·222	·293	·351	·411	·467	·533	·589	·649	·707	·707					
	·000	·003	·018	·042	·071	·104	·140	·143	·209	·222	·293					
21	·137	·213	·276	·338	·398	·455	·506	·551	·602	·662	·724					
	·000	·002	·017	·040	·068	·099	·132	·137	·197	·213	·276					
22	·132	·205	·264	·326	·389	·424	·500	·576	·582	·617	·674	·736				
	·000	·002	·016	·038	·065	·094	·126	·132	·187	·205	·260	·264				
23	·127	·198	·255	·317	·360	·409	·457	·543	·591	·640	·640	·683				
	·000	·002	·016	·037	·062	·090	·120	·127	·178	·198	·247	·255				
24	·122	·191	·246	·308	·347	·396	·443	·500	·557	·604	·653	·661	·692			
	·000	·002	·015	·035	·059	·086	·115	·122	·169	·191	·234	·246	·308			
25	·118	·185	·238	·303	·336	·384	·431	·475	·525	·569	·616	·664	·683			
	·000	·002	·014	·034	·057	·082	·110	·118	·161	·185	·222	·238	·296			
26	·114	·180	·230	·282	·325	·374	·421	·465	·506	·542	·579	·626	·675	·718		
	·000	·002	·014	·032	·054	·079	·106	·114	·154	·180	·212	·230	·282	·282		
27	·110	·175	·223	·269	·316	·364	·415	·437	·500	·563	·570	·598	·636	·684		
	·000	·002	·013	·031	·052	·076	·101	·110	·148	·175	·202	·223	·269	·269		
28	·106	·170	·217	·259	·307	·357	·384	·424	·463	·537	·576	·616	·619	·654	·693	
	·000	·002	·013	·030	·050	·073	·098	·106	·142	·170	·192	·217	·258	·259	·307	
29	·103	·166	·211	·251	·299	·339	·374	·413	·451	·500	·549	·587	·626	·661	·661	
	·000	·002	·012	·029	·049	·070	·094	·103	·136	·166	·184	·211	·247	·251	·299	
30	·100	·163	·205	·244	·292	·324	·364	·403	·440	·476	·524	·560	·597	·636	·676	·676
	·000	·002	·012	·028	·047	·068	·091	·100	·131	·163	·175	·205	·236	·244	·292	·324

TABLE 4 VALUES FOR DETERMINING CONFIDENCE LIMITS FOR AVERAGE COUNT PER ITEM IN A LOT (CONFIDENCE COEFFICIENT 95 PERCENT)

(*Clause* 7.3.2)

[Calculation of sample average count is not necessary. The limits for the average count per item in the lot are obtained by dividing the values in col 2 and 3 by the number *n* of items in the sample.]

SUM OF COUNT ON *n* ITEMS (1)	*n* TIMES LOWER LIMIT (2)	*n* TIMES UPPER LIMIT (3)
0	0·000 0	3·69
1	0·025 3	5·57
2	0·242	7·22
3	0·619	8·77
4	1·09	10·24
5	1·62	11·67
6	2·20	13·06
7	2·81	14·42
8	3·45	15·76
9	4·12	17·08
10	4·80	18·39
11	5·49	19·68
12	6·20	20·96
13	6·92	22·23
14	7·65	23·49
15	8·40	24·74
16	9·15	25·98
17	9·90	27·22
18	10·67	28·45
19	11·44	29·67
20	12·22	30·89
21	13·00	32·10
22	13·79	33·31
23	14·58	34·51
24	15·38	35·71
25	16·18	36·90
26	16·98	38·10
27	17·79	39·28
28	18·61	40·47
29	19·42	41·65
30	20·24	42·83
35	24·38	48·68
40	28·58	54·47
45	32·82	60·21
50	37·11	65·92

TABLE 5 VALUES OF h' FOR DETERMINING CONFIDENCE LIMITS FOR THE AVERAGE (ARITHMETIC MEAN) OF A MEASURED CHARACTERISTIC IN THE LOT, USING SAMPLE MEAN WITH SAMPLE STANDARD DEVIATION (CONFIDENCE COEFFICIENT 95 PERCENT)

(*Clause* 7.3.3)

[The lower and upper confidence limits are given by ($\bar{x} - h's$) and ($\bar{x} + h's$) respectively].

NUMBER OF ITEMS IN THE SAMPLE n	VALUE OF h'	NUMBER OF ITEMS IN THE SAMPLE n	VALUE OF h'
2	8·984	22	0·443
3	2·484	23	0·432
4	1·591	24	0·422
5	1·242	25	0·413
6	1·050	26	0·404
7	0·925	27	0·396
8	0·836	28	0·388
9	0·769	29	0·380
10	0·715	30	0·373
11	0·672	31	0·367
12	0·635	41	0·316
13	0·604	51	0·281
14	0·577	61	0·256
15	0·554	81	0·221
16	0·533	101	0·198
17	0·514	121	0·180
18	0·497	More than 121, use formula:	$\frac{1·96}{\sqrt{n}}$
19	0·482		
20	0·468		
21	0·455		

TABLE 6 VALUES OF *h* FOR DETERMINING CONFIDENCE LIMITS FOR THE AVERAGE (ARITHMETIC MEAN) OF A MEASURED CHARACTERISTIC IN THE LOT, USING SAMPLE MEAN AND SAMPLE RANGE (OR MEAN RANGE) (CONFIDENCE COEFFICIENT 95 PERCENT)

(*Clause* 7.3.3)

[The lower and upper confidence limits are given ($\bar{x} - hR$) and ($\bar{x} + hR$) where R is based on a gross sample of n items (that is, $m = 1$) and ($\bar{x} - h\bar{R}$) and ($\bar{x} + h\bar{R}$) where $\bar{R}$ is the average of m ranges from m sub-samples each of size n.]

NUMBER OF ITEMS IN THE SAMPLE *n*	*m* = NUMBER OF SUB-SAMPLE EACH OF SIZE *n*									
	1	2	3	4	5	6	7	8	9	10
2	6·36	1·72	1·08	0·83	0·70	0·61	0·55	0·50	0·46	0·44
3	1·30	0·64	0·47	0·38	0·33	0·30	0·27	0·25	0·24	0·22
4	0.72	0·41	0·31	0·26	0·23	0·21	0·19	0·18	0·17	0·16
5	0·51	0·31	0·24	0·20	0·18	0·16	0·14	0·15	0·13	0·12
6	0·40	0·25	0·20	0·17	0·15	0·13	0·12	0·11	0·11	0·10
7	0·33	0·21	0·17	0·14	0·13	0·12	0·11	0·10	0·09	0·09
8	0·29	0·19	0·15	0·13	0·11	0·10	0·09	0·09	0·08	0·08
9	0·25	0·17	0·13	0·11	0·10	0·09	0·08	0·08	0·07	0·07
10	0·23	0·15	0·12	0·10	0·09	0·08	0·08	0·07	0·07	0·06
11	0·21	0·14	0·11	0·10	0·08	0·07	0·07	0·07	0·06	0·06
12	0·19	0·13	0·10	0·09	0·08	0·07	0·07	0·06	0·06	0·06
13	0·18	0·12	0·10	0·08	0·07	0·07	0·06	0·06	0·05	0·05
14	0·17	0·11	0·09	0·08	0·07	0·06	0·06	0·05	0·05	0·05
15	0·16	0·11	0·09	0·07	0·07	0·06	0·06	0·05	0·05	0·05

TABLE 7 FACTORS f_1 AND f_2 FOR DETERMINING CONFIDENCE LIMITS FOR VARIABILITY (STANDARD DEVIATION) OF A MEASURED CHARACTERISTIC IN THE LOT , USING SAMPLE STANDARD DEVIATION (CONFIDENCE COEFFICIENT 95 PERCENT)

(*Clause* 7.4.1)

[The lower and upper limits are given by f_1s and f_2s, where s is the sample standard deviation based on a sample of n items.]

NUMBER OF ITEMS IN THE SAMPLE n	VALUE OF f_1	VALUE OF f_2
2	0·446	31·91
3	0·521	6·28
4	0·566	3·73
5	0·599	2·87
6	0·624	2·45
7	0·644	2·20
8	0·661	2·04
9	0·675	1·92
10	0·688	1·83
11	0·699	1·75
12	0·708	1·70
13	0·717	1·65
14	0·725	1·61
15	0·732	1·58
16	0·739	1·55
17	0·745	1·52
18	0·750	1·50
19	0·756	1·48
20	0·760	1·46
21	0·765	1·44
26	0·784	1·38
31	0·799	1·34
36	0·811	1·30
41	0·821	1·28
46	0·829	1·26
51	0·837	1·24
61	0·849	1.22
71	0·858	1·20
81	0·866	1·18
91	0·873	1·17
101	0·879	1·16
For values 102 and above, use the approximate formula:	$1 - \frac{1{\cdot}96}{\sqrt{2n}}$ and	$1 + \frac{1{\cdot}96}{\sqrt{2n}}$

he lower and upper limits are given by f_3R and f_4R, where R is the sample range based on a sample of n items.]

UMBER OF ITEMS N THE SAMPLE n	VALUE OF f_3	VALUE OF f_4
2	0·315	22·3
3	0·272	3·30
4	0·251	1·68
5	0·238	1·18
6	0·229	0·938
7	0·223	0·799
8	0·217	0·709
9	0·213	0·645
10	0·209	0·597
11	0·206	0·561
12	0·203	0·531
13	0·201	0·506
14	0·198	0·486
15	0·196	0·468
16	0·195	0·453
17	0·193	0·440
18	0·191	0·428
19	0·190	0·418
20	0·189	0·408

BUREAU OF INDIAN STANDARDS

Headquarters:

Manak Bhavan, 9 Bahadur Shah Zafar Marg, NEW DELHI 110002
Telephones: 323 0131, 323 3375, 323 9402 Fax :+ 91 011 3234062, 3239399, 3239382
E - Mail : bis@vsnl.com **Website** : http://www.bis.org.in

Central Laboratory:	Telephone
Plot No. 20/9, Site IV, Sahibabad Industrial Area, Sahibabad 201010	477 00 32
Regional Offices:	
Central : Manak Bhavan, 9 Bahadur Shah Zafar Marg, NEW DELHI 110002	323 76 17
*Eastern : 1/14 CIT Scheme VII, V.I.P. Road, Kankurgachi, CALCUTTA 700054	337 86 62
Northern : SCO 335-336, Sector 34-A, CHANDIGARH 160022	60 38 43
Southern : C.I.T. Campus, IV Cross Road, CHENNAI 600113	254 13 15
†Western : Manakalaya, E9, MIDC, Behind Marol Telephone Exchange, Andheri (East), MUMBAI 400093	832 92 95
Branch Offices:	
'Pushpak', Nurmohamed Shaikh Marg, Khanpur, AHMEDABAD 380001	550 13 48
Peenya Industrial Area, 1st Stage, Bangalore-Tumkur Road, BANGALORE 560058	839 49 55
Commercial-cum-Office Complex, Opp. Dushera Maidan, E-5 Arera Colony, Bittan Market, BHOPAL 462016	72 34 52
62-63, Ganga Nagar, Unit VI, BHUBANESHWAR 751001	40 36 27
5th Floor, Kovai Towers, 44 Bala Sundaram Road, COIMBATORE 641018	21 88 35
Plot No. 58, Neelam Bata Road, NIT, FARIDABAD 121001	542 82 61
Savitri Complex, 116 G.T. Road, GHAZIABAD 201001	471 19 98
53/5 Ward No. 29, R.G. Barua Road, 5th By-lane, Apurba Sinha Path, GUWAHATI 781003	54 11 37
5-8-56C, L.N. Gupta Marg, Nampally Station Road, HYDERABAD 500001	320 10 84
E-52, Chitaranjan Marg, C-Scheme, JAIPUR 302001	37 38 79
117/418 B, Sarvodaya Nagar, KANPUR 208005	21 68 76
Seth Bhawan, 2nd Floor, Behind Leela Cinema, Naval Kishore Road, LUCKNOW 226001	21 89 23
NIT Building, Second Floor, Gokulpat Market, NAGPUR 440010	52 51 71
Mahabir Bhawan, 1st Floor, Ropar Road, NALAGARH 174101	2 14 51
Patliputra Industrial Estate, PATNA 800013	26 28 08
First Floor, Plot Nos. 657-660, Market Yard, Gultekdi, PUNE 411037	426 86 59
'Sahajanand House' 3rd Floor, Bhaktinagar Circle, 80 Feet Road, RAJKOT 360002	37 82 51
T.C. No. 14/1421, University P.O. Palayam, THIRUVANANTHAPURAM 695034	32 21 04

---

*Sales Office is at 5 Chowringhee Approach, P.O. Princep Street, CALCUTTA 700072	237 10 85
†Sales Office is at Novelty Chambers, Grant Road, MUMBAI 400007	309 65 28

Printed at Simco Printing Press, Delhi